Studying the Multi-Scale Dynamics of the Oceanic Symphony

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The Ocean is Like a Symphony

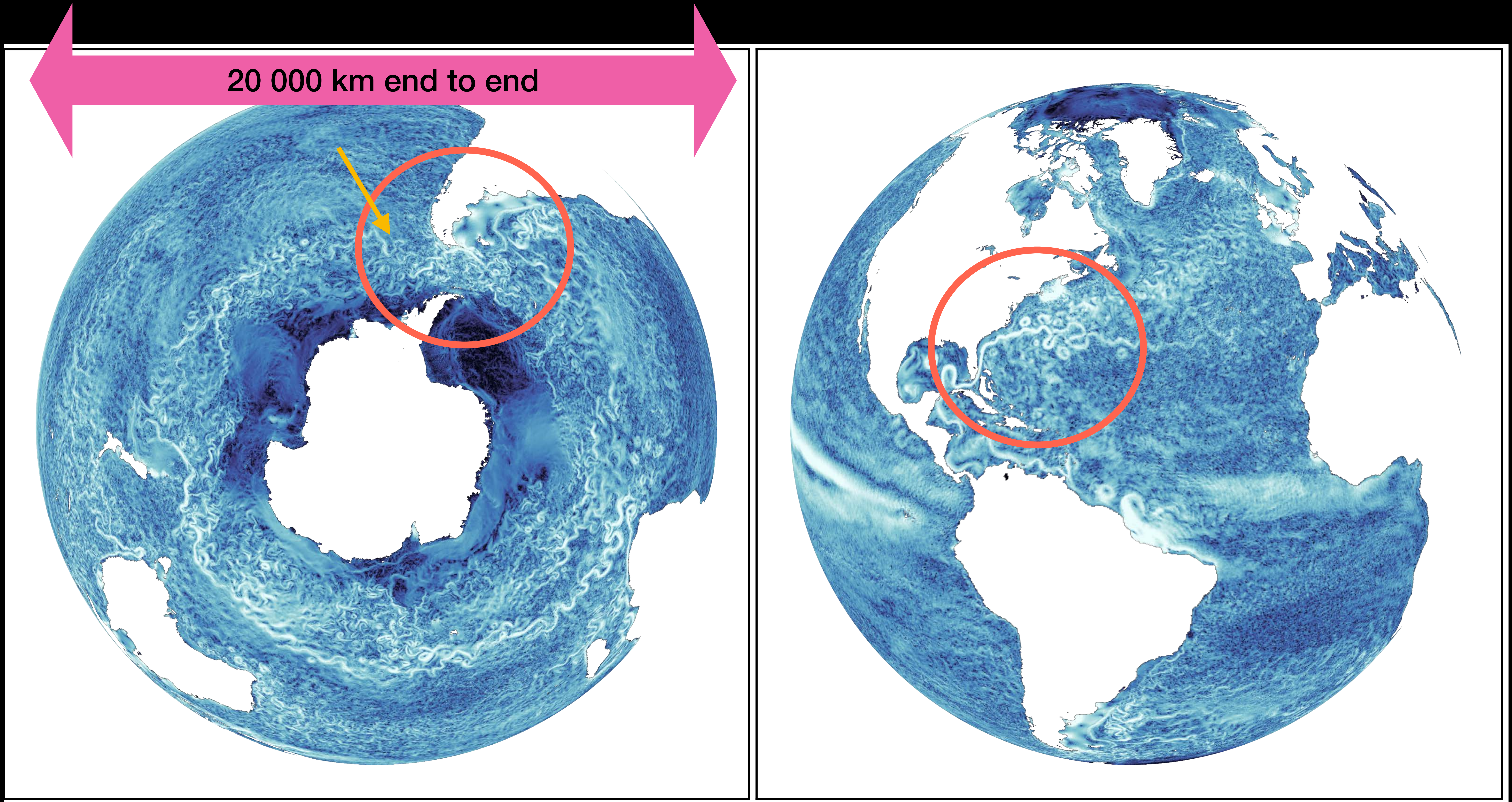
- A symphony is a constantly changing balance of different tones
  - Many notes are sounding at once, but we hear them as a cohesive whole
  - We can analyze each chord and identify individual notes (pitch/frequency) and their volume (energy)
  - Sounds can interact with each other to produce beating patterns at a new frequency
  - Some notes are very short, while others have long durations

The Ocean is Like a Symphony

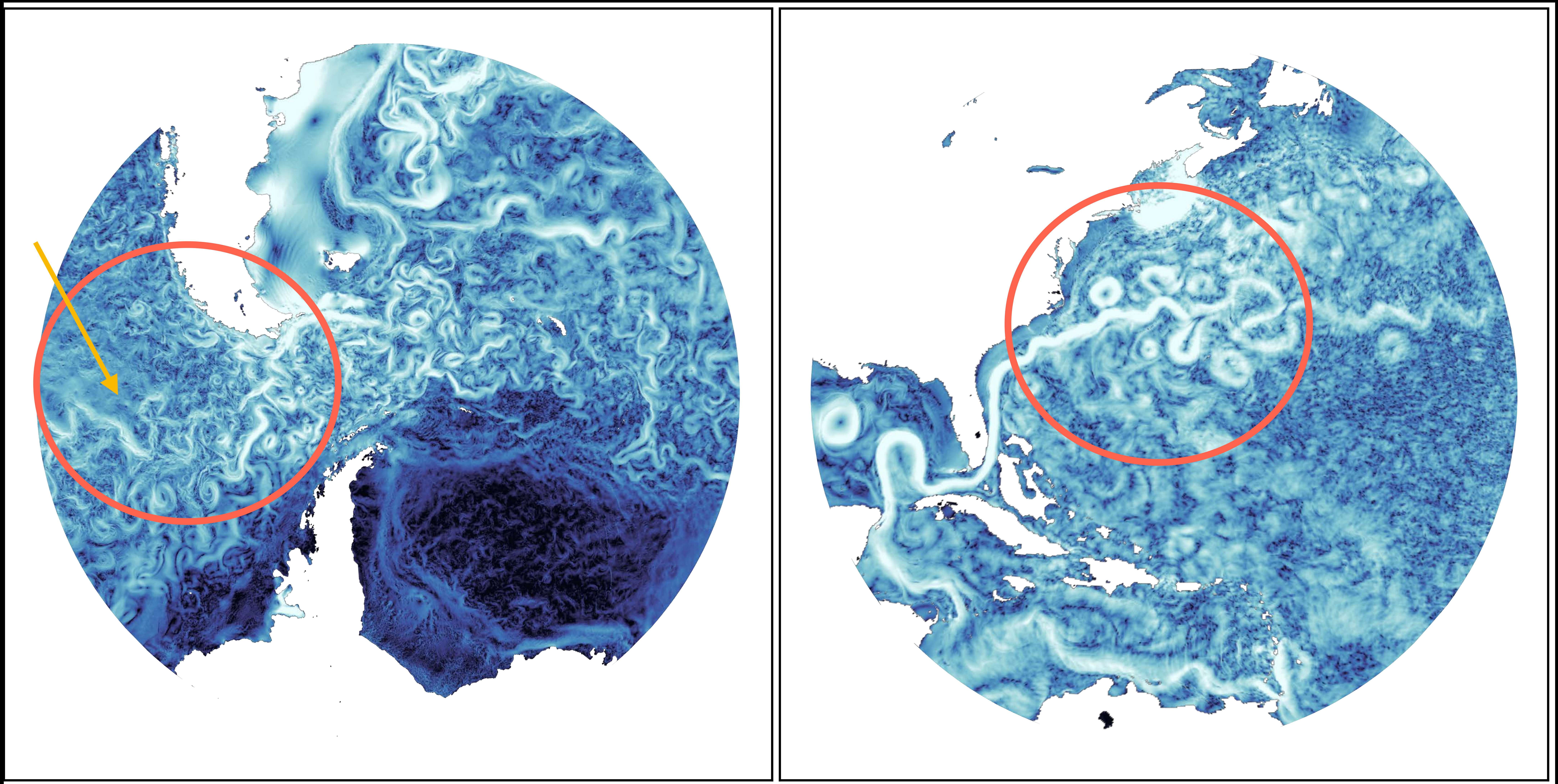
- The circumference of the Earth (largest scale [lowest note]) is
  $\sim 40\ 000\ \mathrm{km} = 4 \times 10^7\ \mathrm{m}$
- Dissipation of kinetic energy into heat at very small scales [highest notes],
  $\sim 1\ \mathrm{mm} = 10^{-3}\ \mathrm{m}$
- Over 10 orders of magnitude [35 octaves] between largest and smallest scales!
  - For reference, the sun is 6 orders of magnitude more massive than Earth

A snapshot of speed of ocean currents

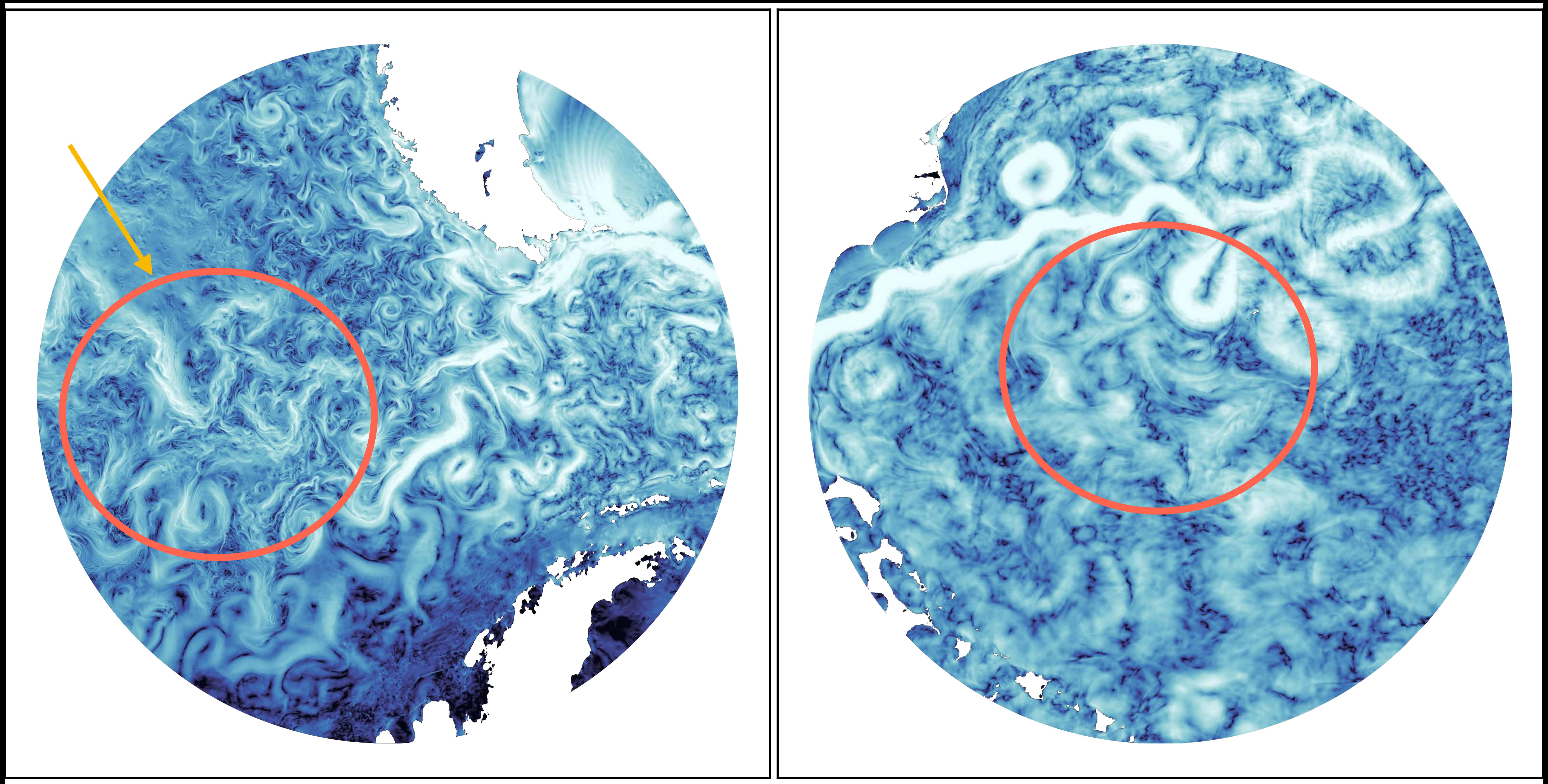
[MITGCM's LLC4320]



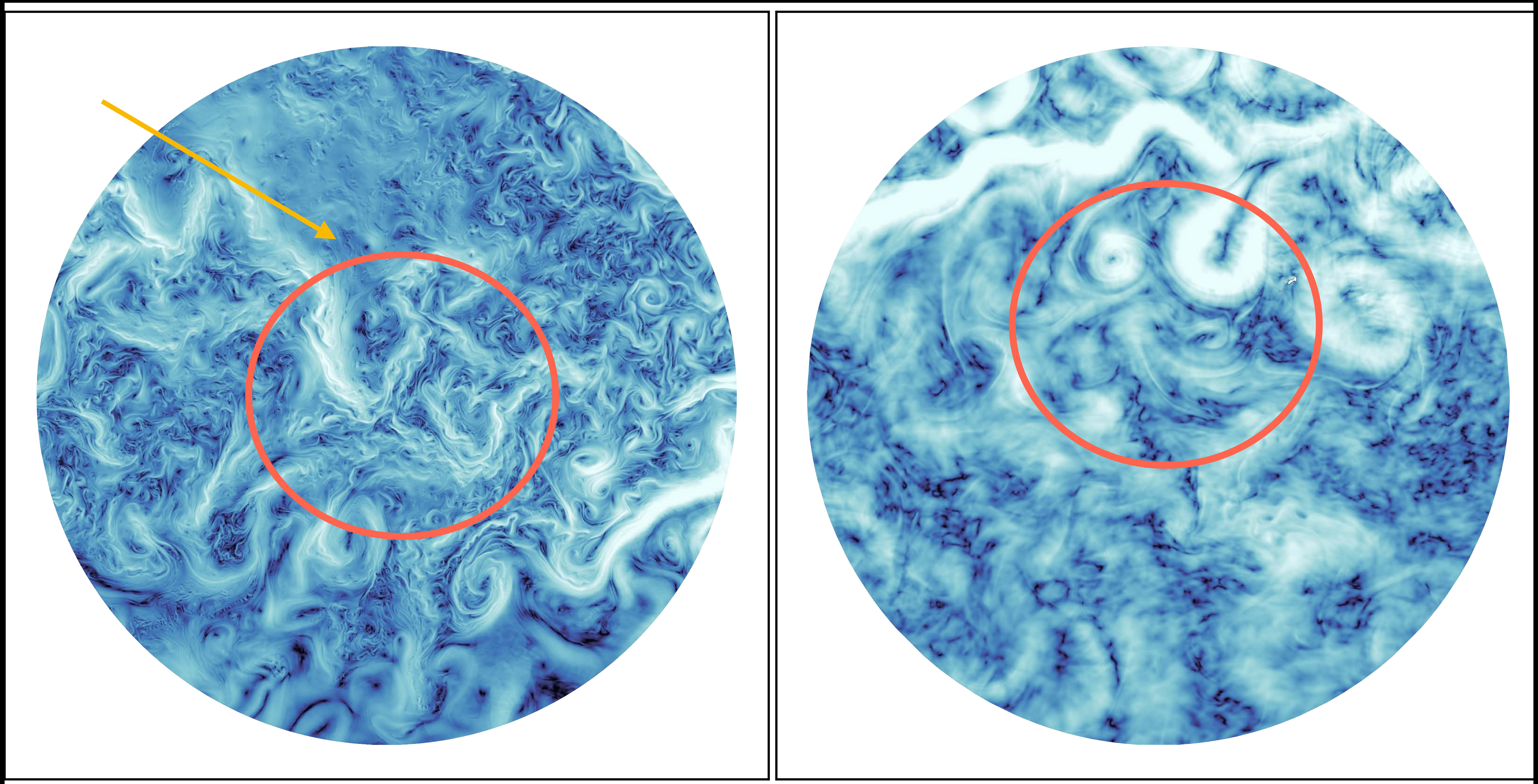
Zooming in…



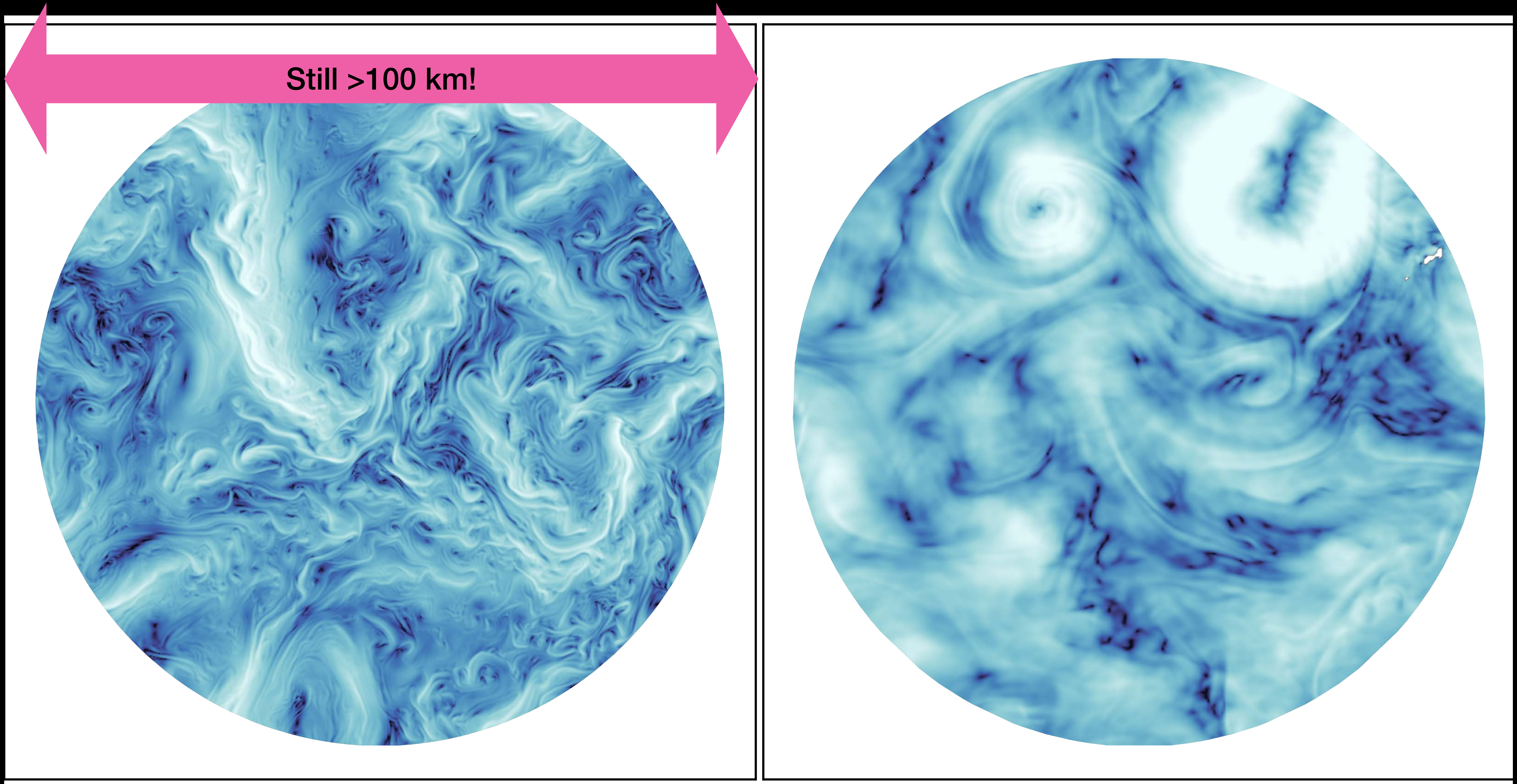
Zooming further…



Zooming even further still…



And there are still smaller scales!



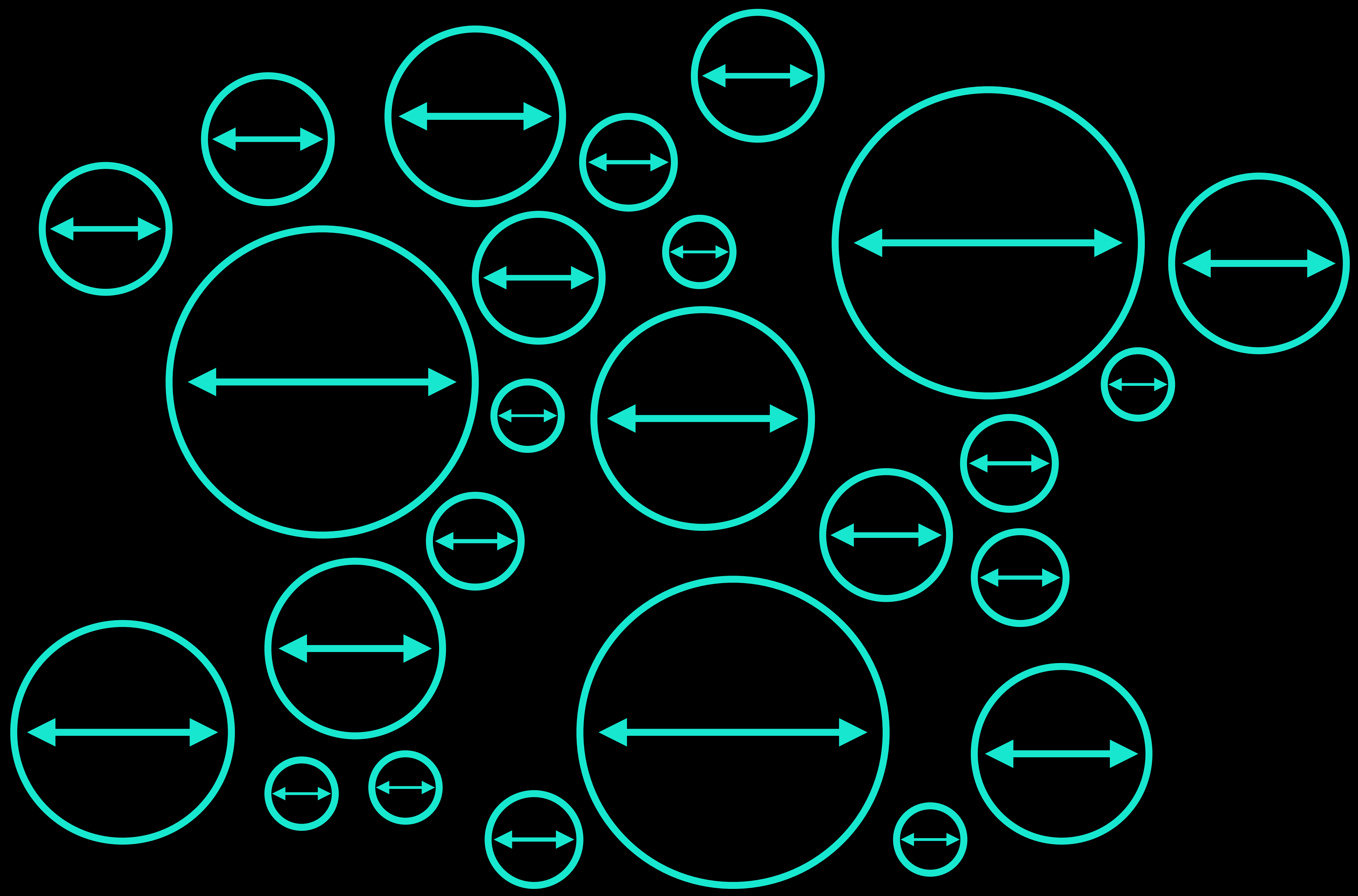
The Ocean is Like a Symphony

- The oceanic symphony is dynamically rich, with interesting and meaningful behaviour at a wide range of scales

- We would like to decompose the ocean flow by length-scale, to study the nature of, and interaction between, these scales

  - e.g. how loud is each note?

  - how does that energy change over time?



Scale Decomposition

- There are many methods for decomposing a signal into constituent scales
  - Fourier Methods
    - Decomposes a signal into sin/cosine terms. Requires flat/Cartesian geometries
  - Spherical Harmonics
    - Analog of Fourier Methods for spherical geometries
  - Coarse-Graining
    - Method presented here: a generalized scale-decomposition routine that is geometry-agnostic
  - Reynolds Averaging
    - Not truly a spatial decomposition, instead divides into time-mean and time-varying parts

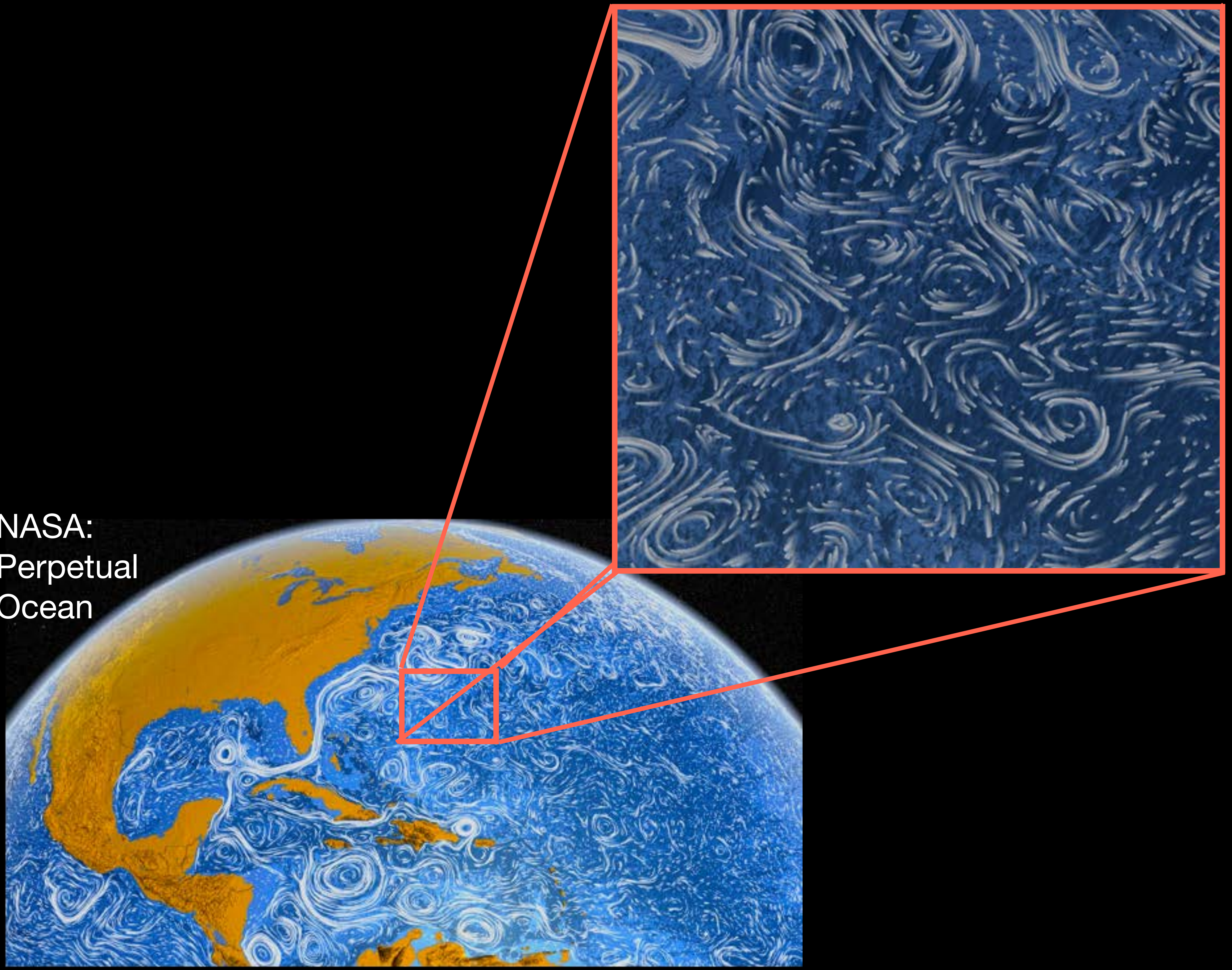
Scale Decomposition: Traditional Approach

Using Fourier on the Globe

- Traditional Fourier transforms only work on flat surfaces
  - *i.e. not spheres (like the Earth)*
  - So we'll pick a small box that is "flat enough"

NASA: Perpetual Ocean

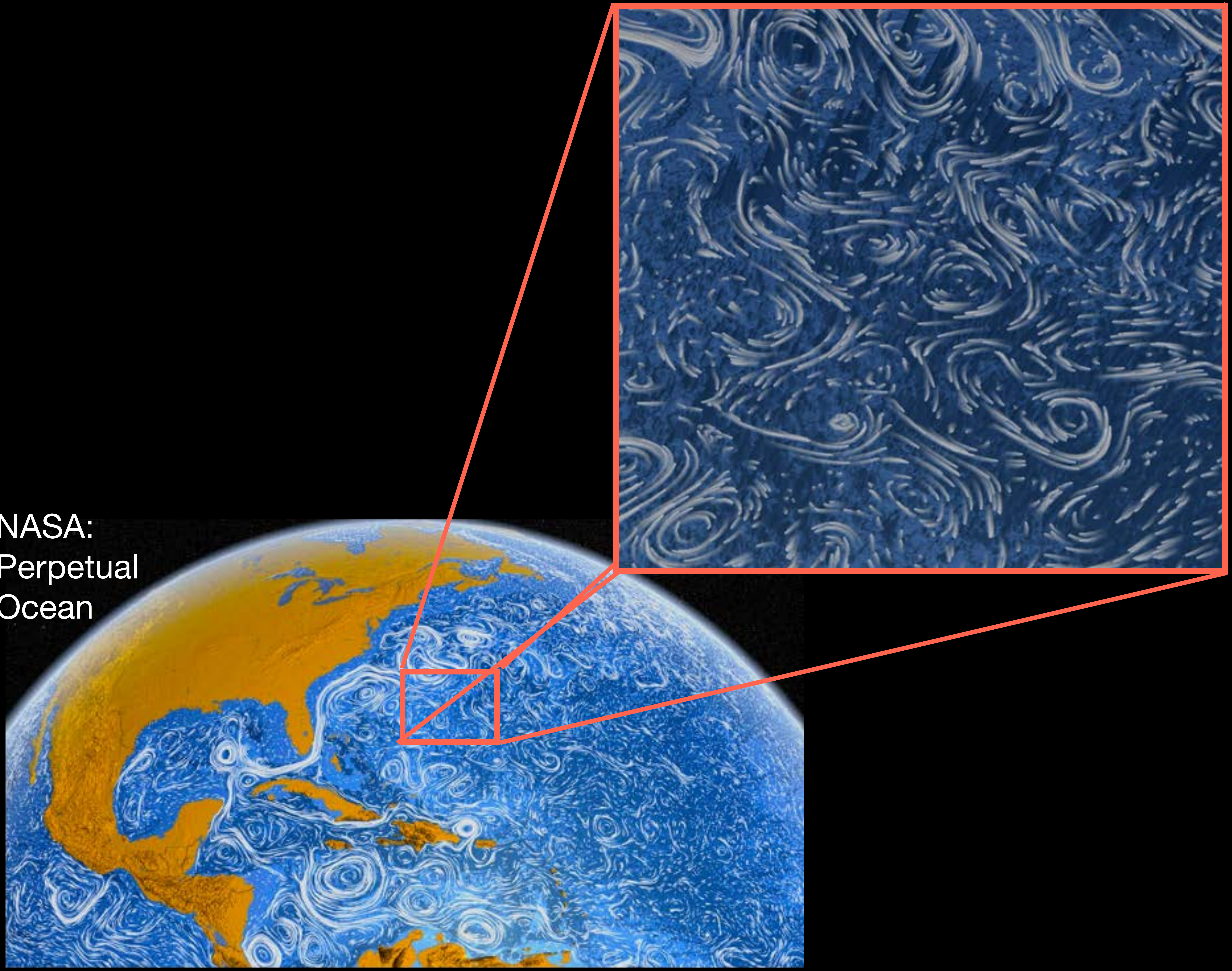
Scale Decomposition: Traditional Approach



NASA: Perpetual Ocean

- But there's no reason that the stuff inside the box is periodic
- So we'll apply an envelope to make it periodic

Scale Decomposition: Traditional Approach



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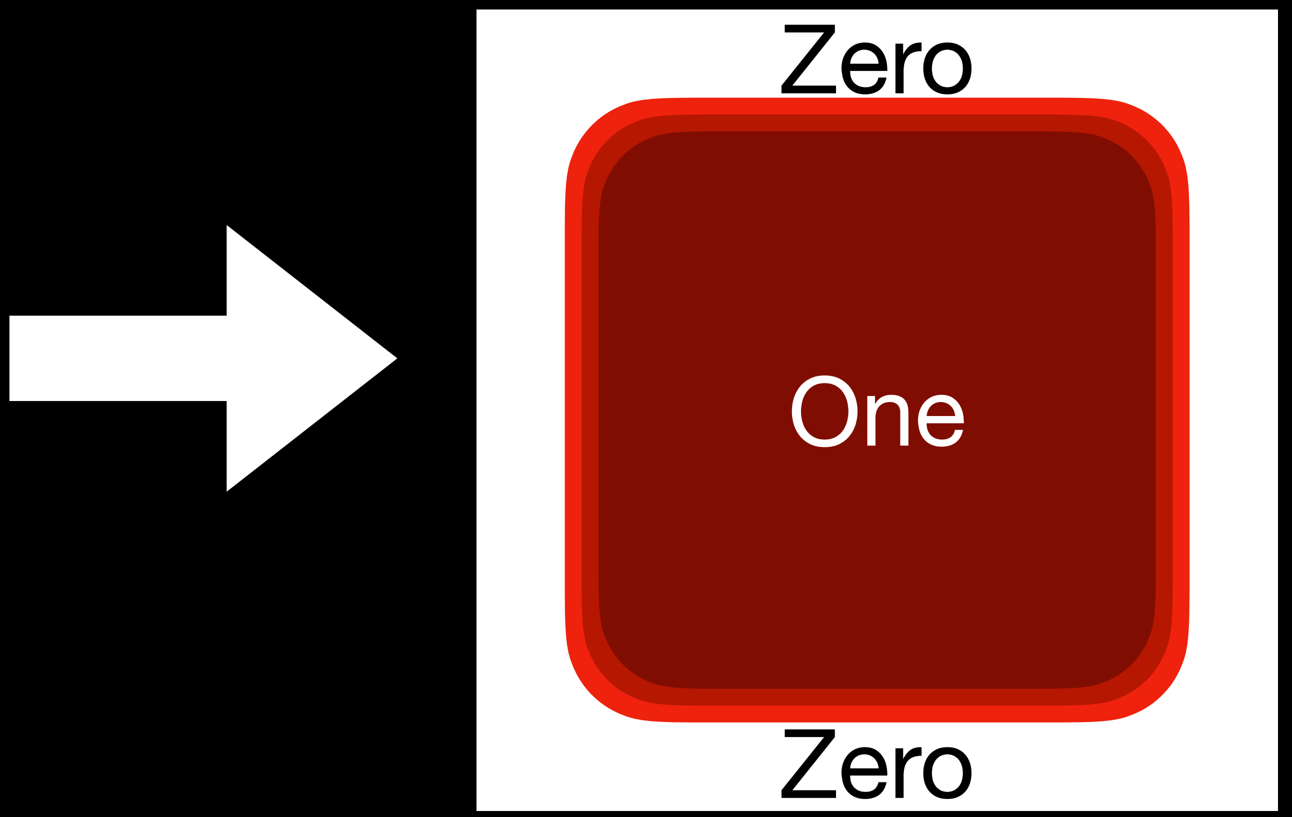

- But now we’ve contaminated the large scales in the box

Traditional Approach

- Fourier Methods have allowed many great insights into ocean energy dynamics

- Detailed analysis of ocean KE spectra

  Fu and Smith (1996), Chen et al. (2015), Rocha et al. (2016), Khatri et al. (2018), O'Rourke et al. (2018), Callies and Wu (2019)

- Provided insight into length-scales of motion and cascades through them

  Scott and Wang (2005), Scott and Arbic (2007), Arbic et al (2012, 2013, 2014)

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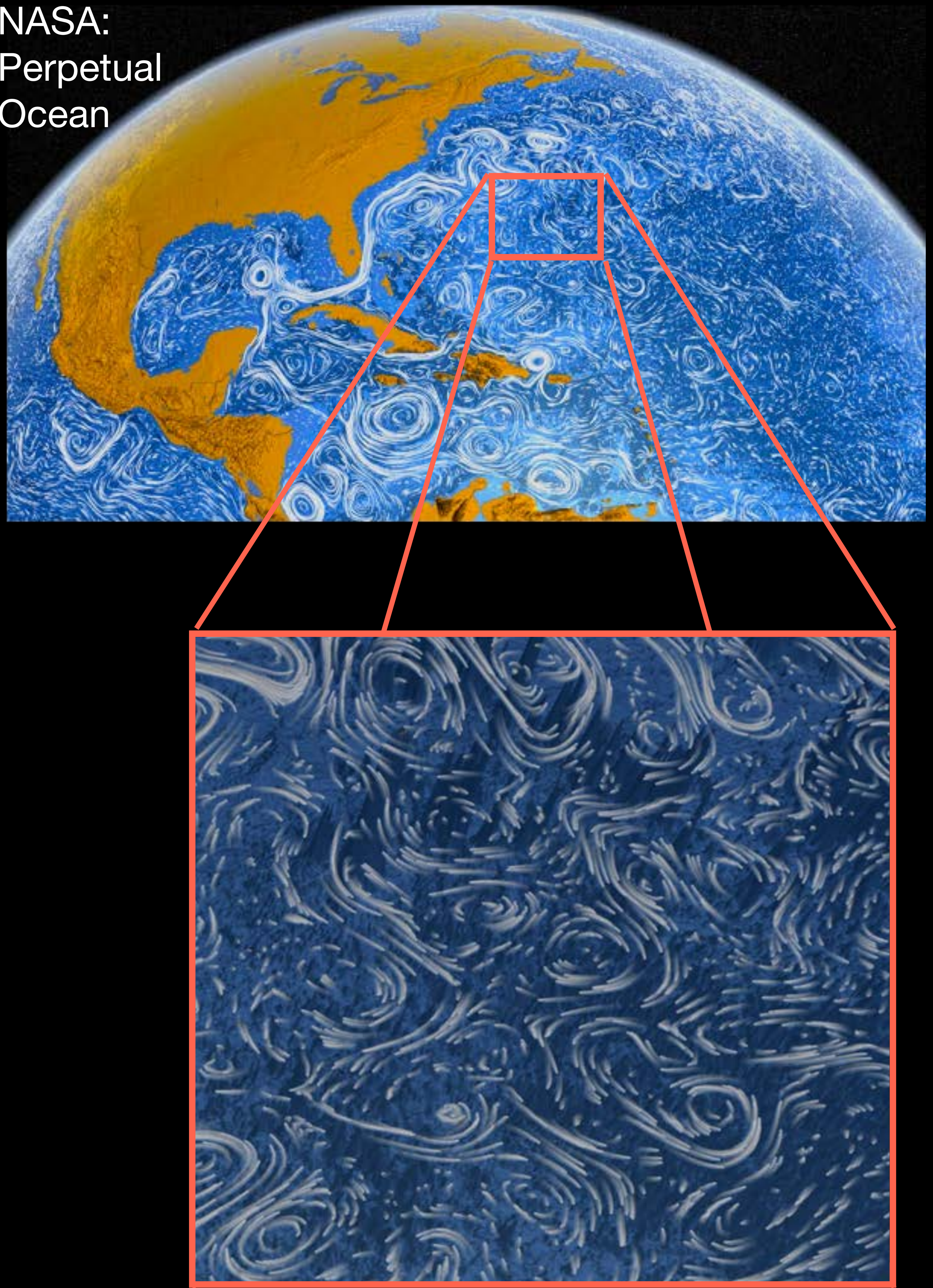

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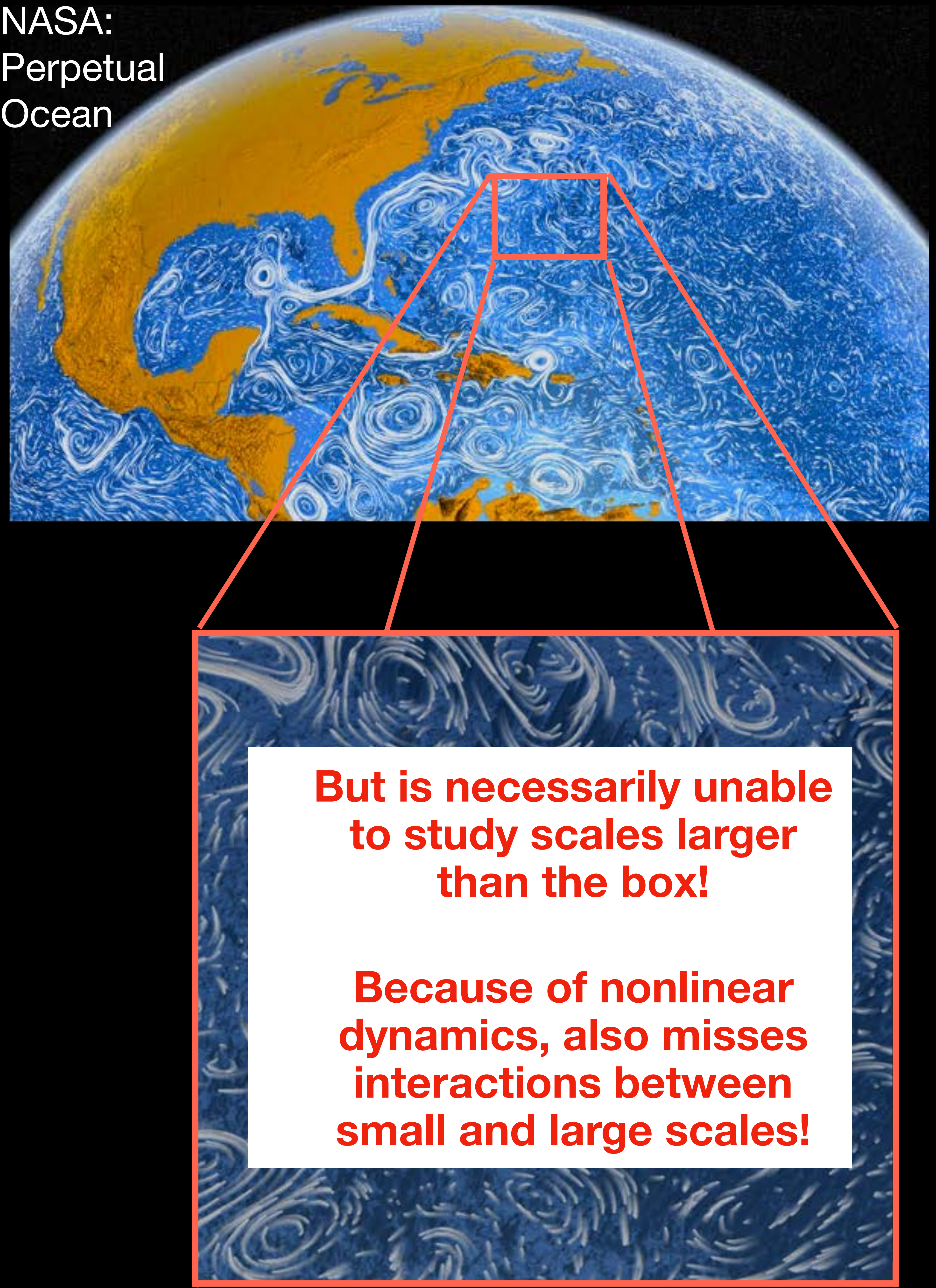
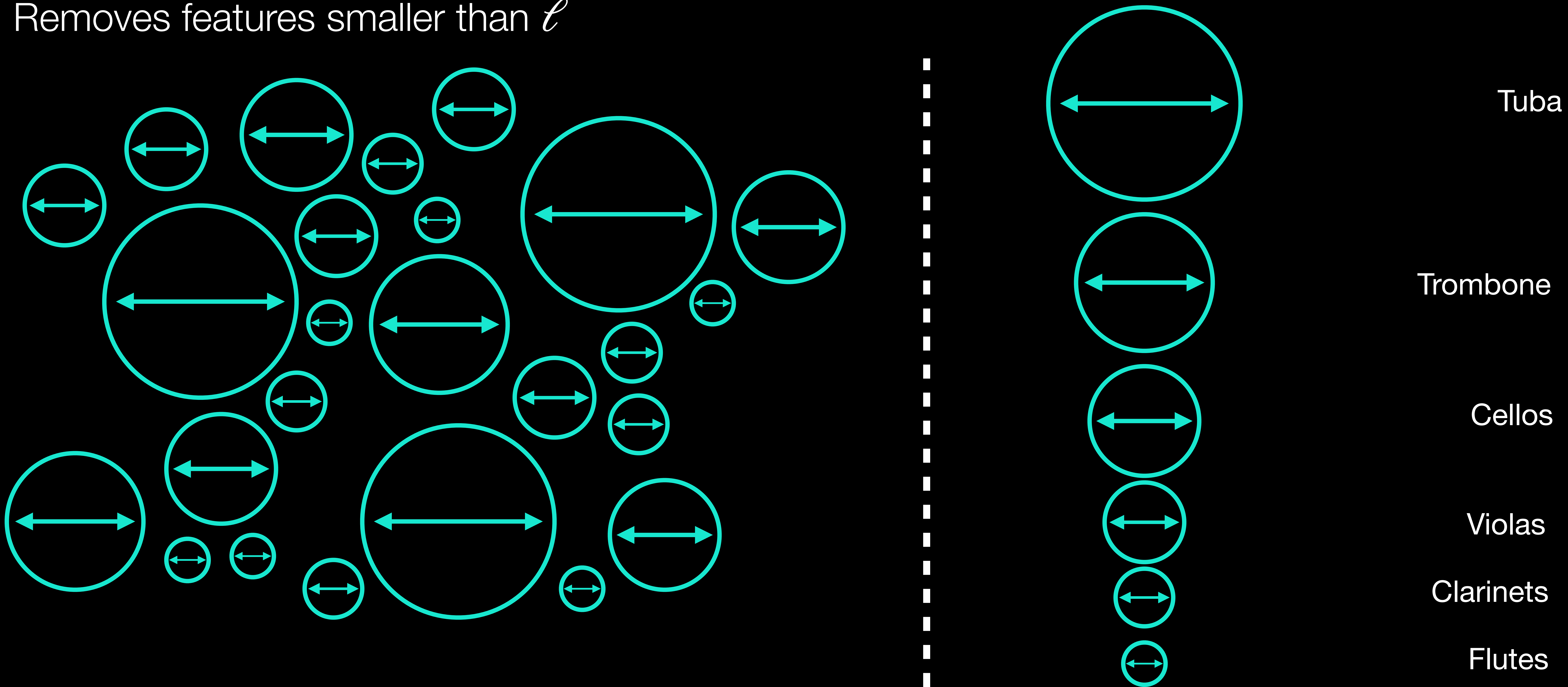


Coarse-graining

- Choose a length scale ℓ (in metres), and smooth / blur the fields. Essentially a locally weighted moving average in space
- Removes features smaller than ℓ



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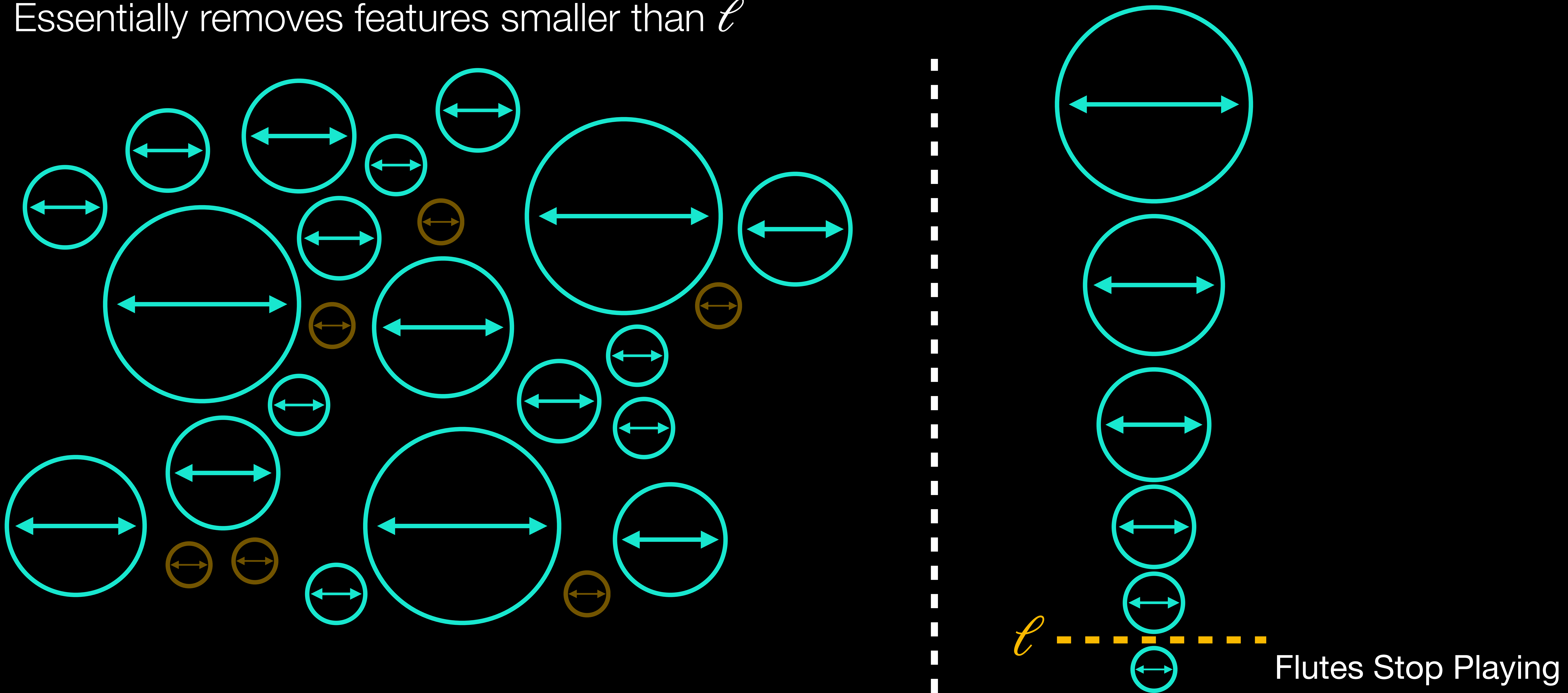

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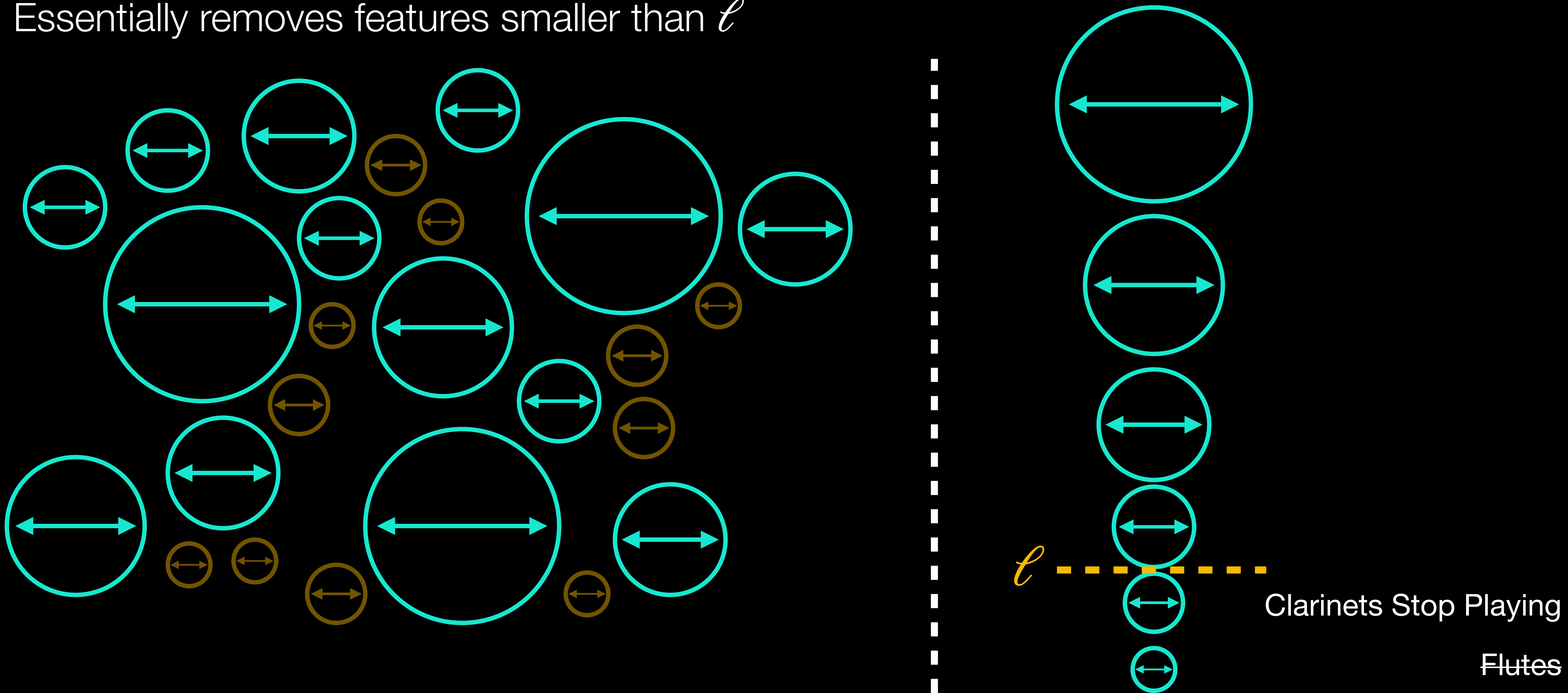

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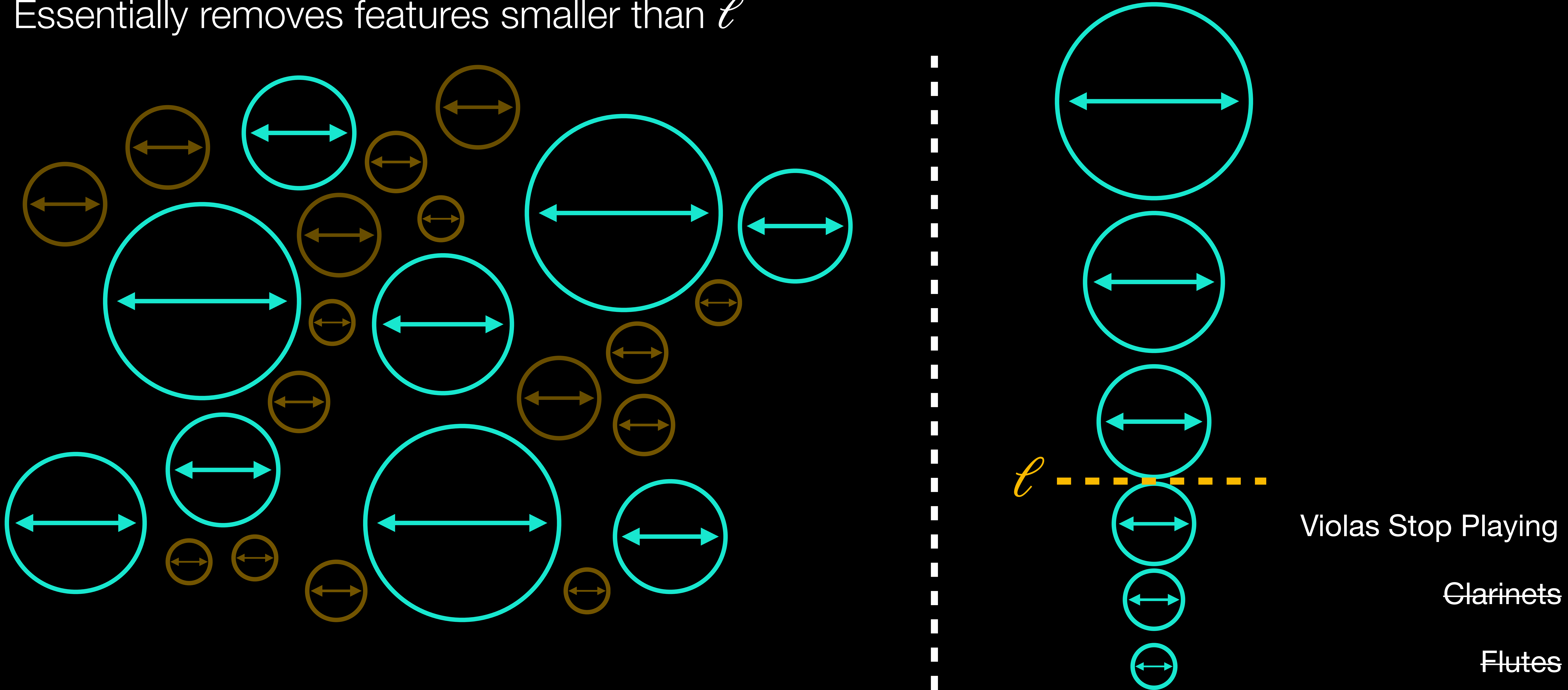

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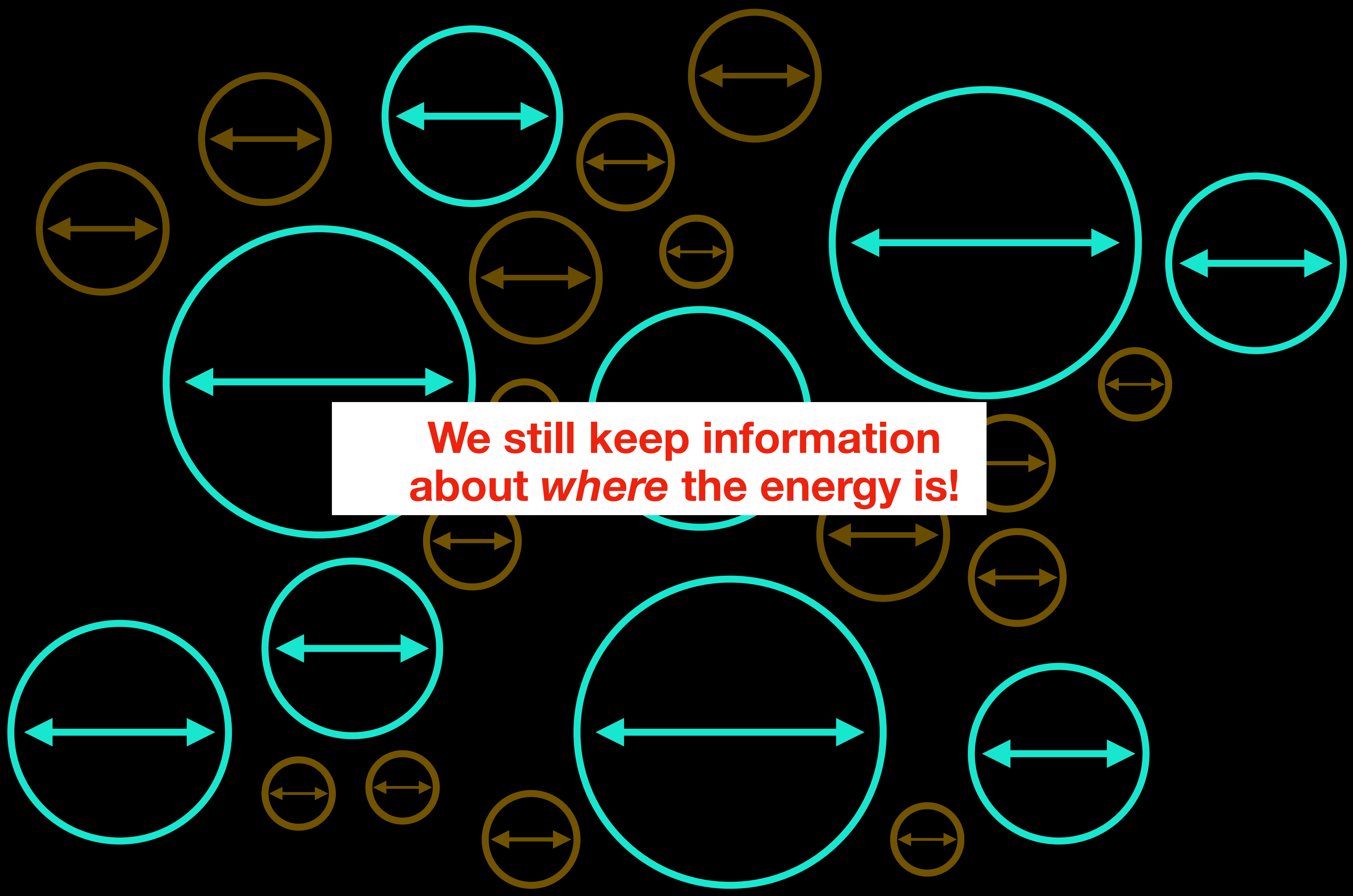

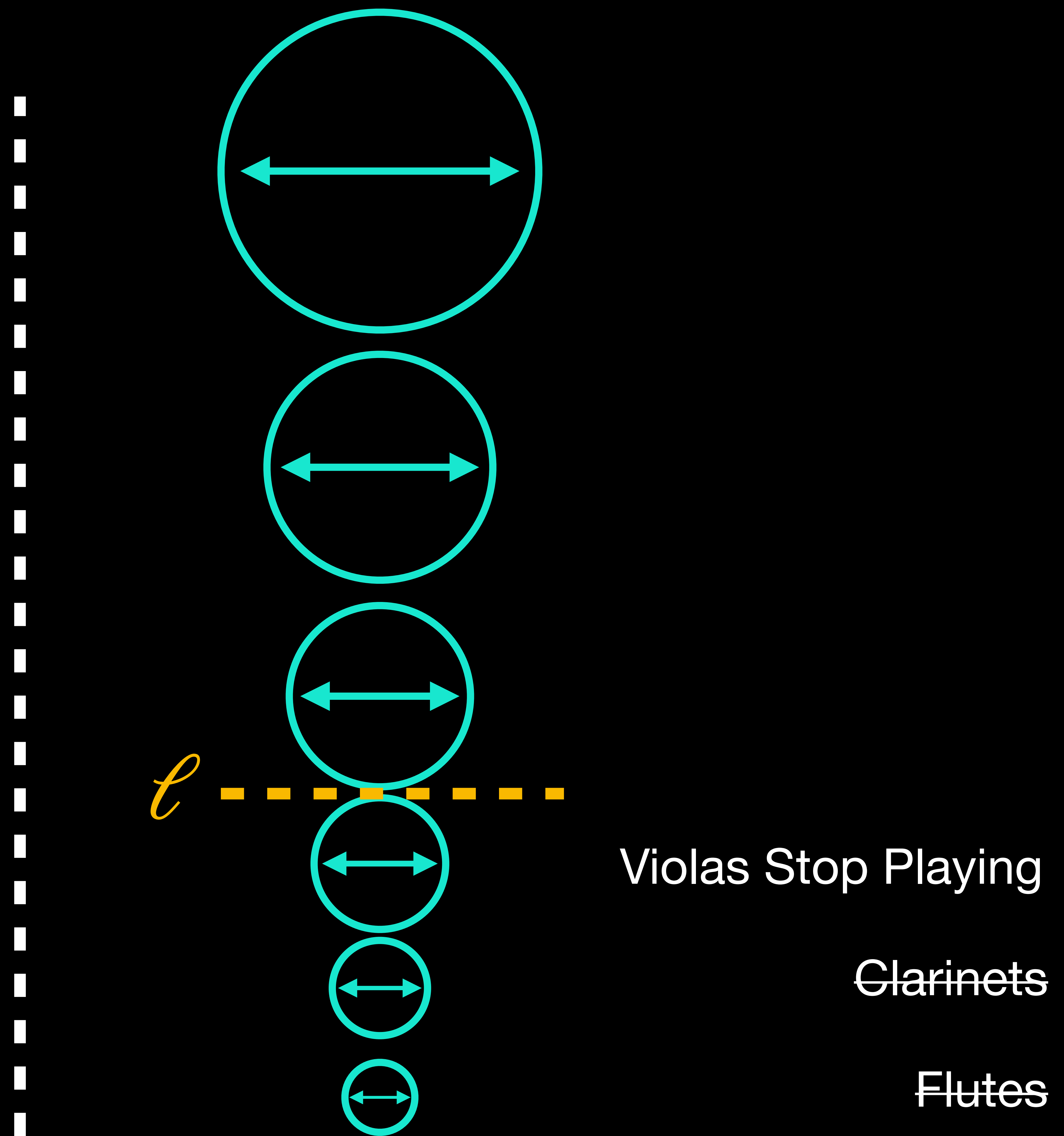

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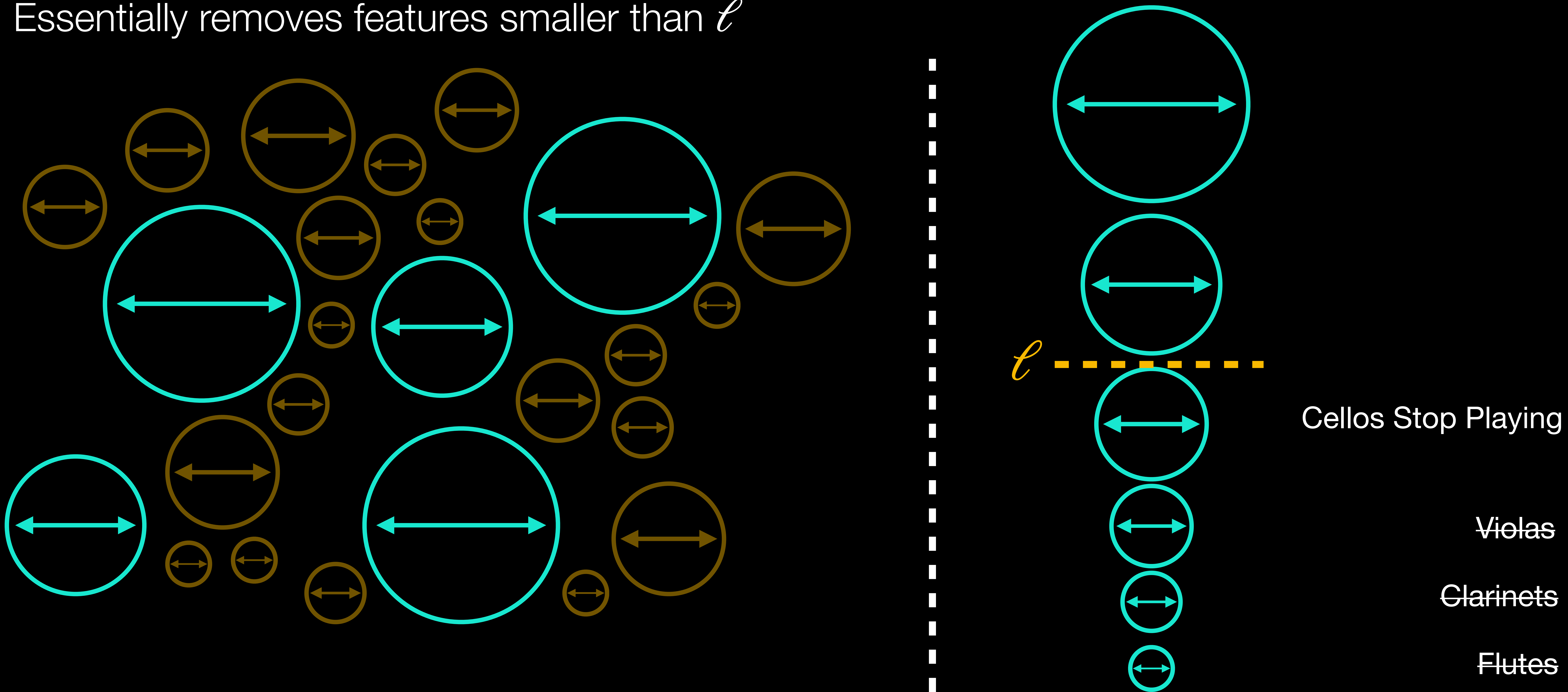

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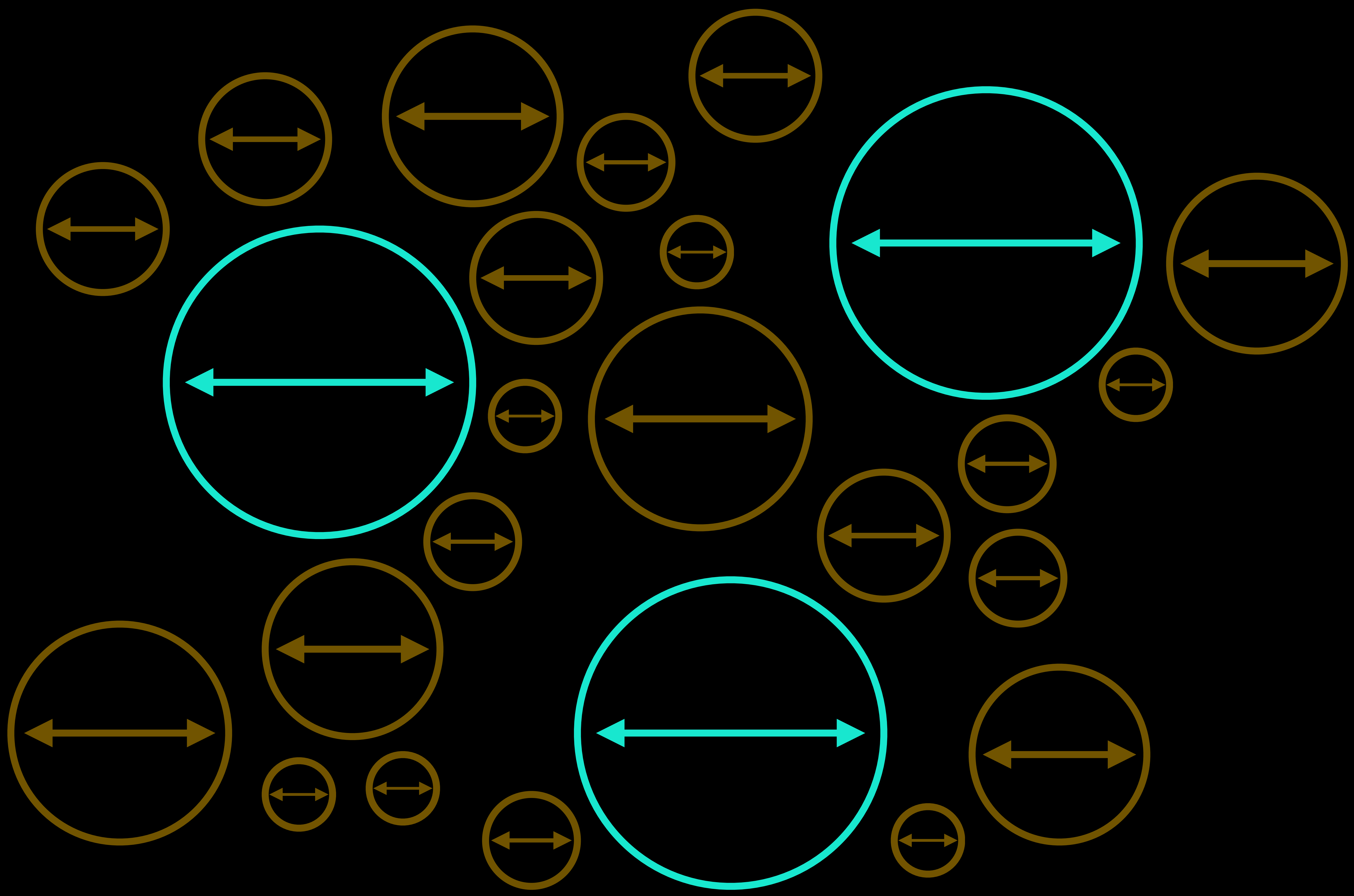

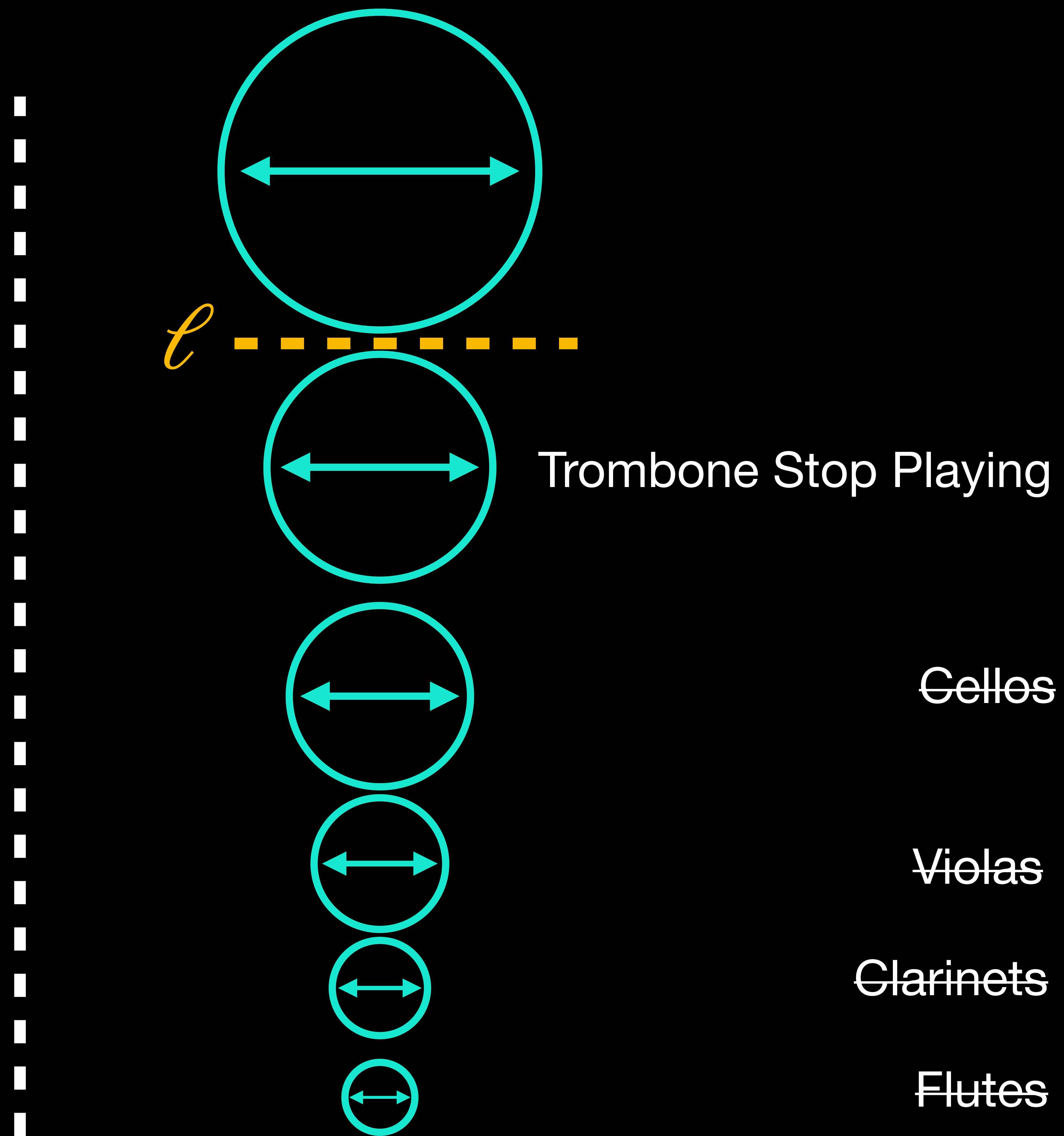

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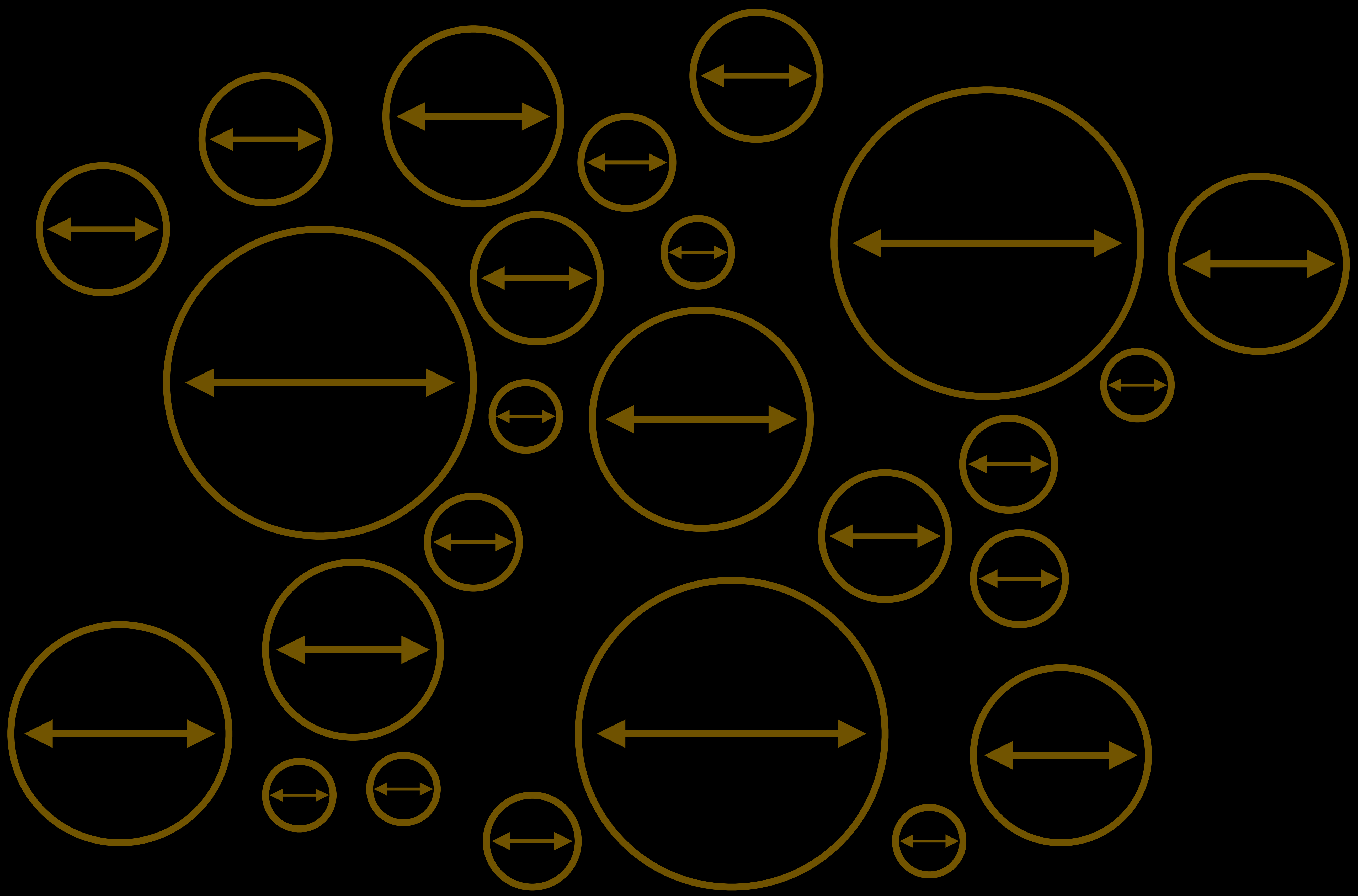
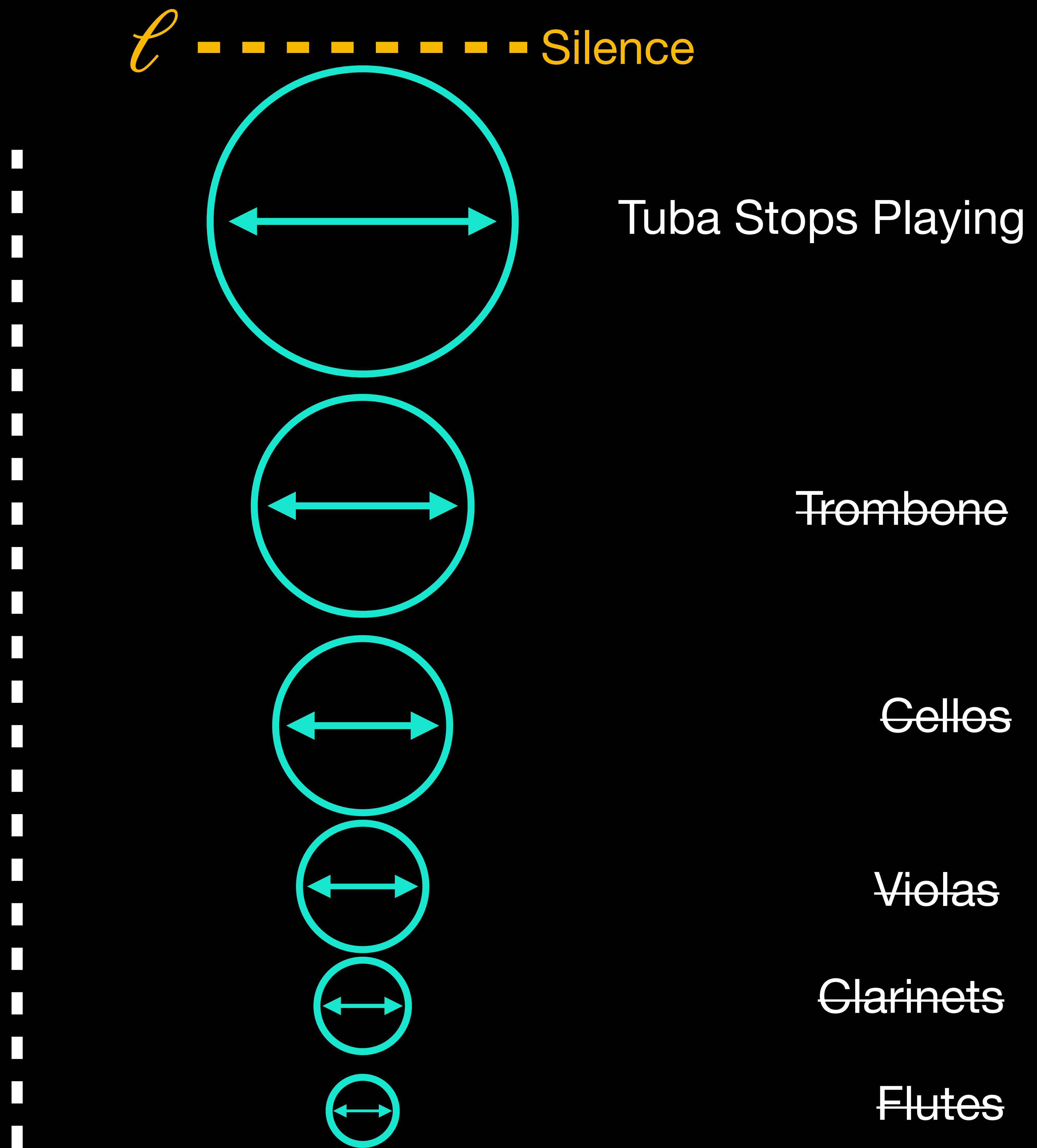
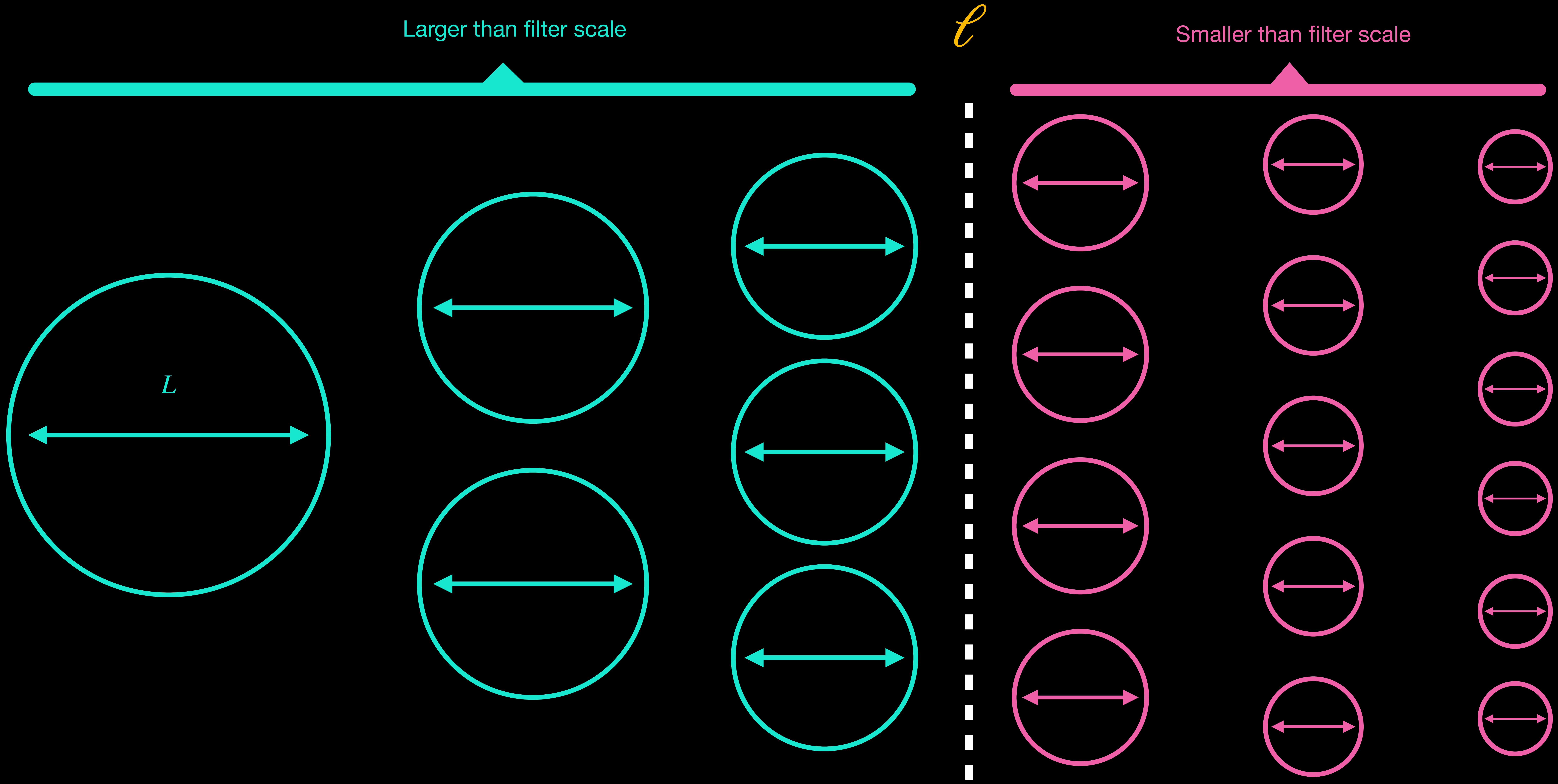




Coarse-graining: Measuring the Energy



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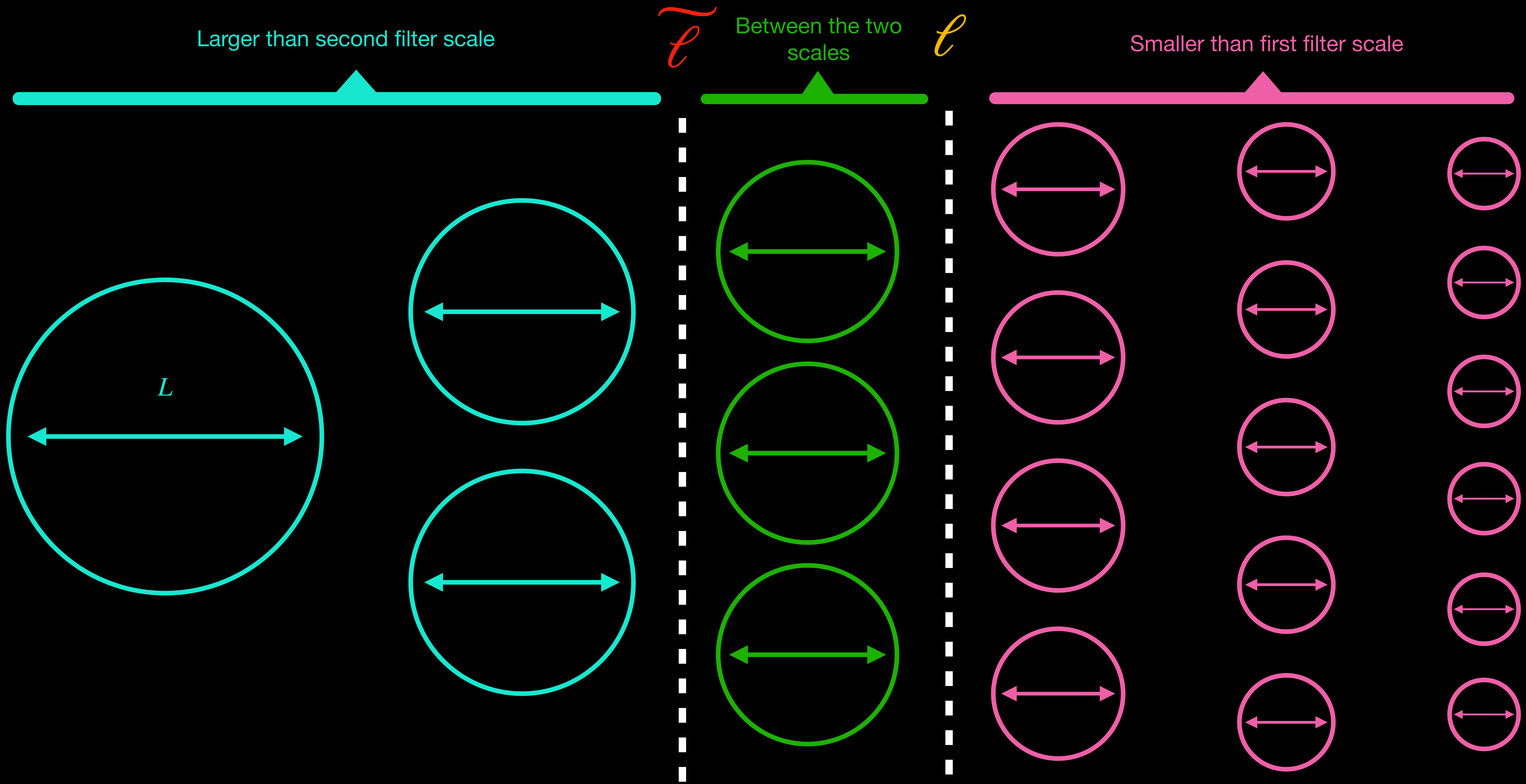

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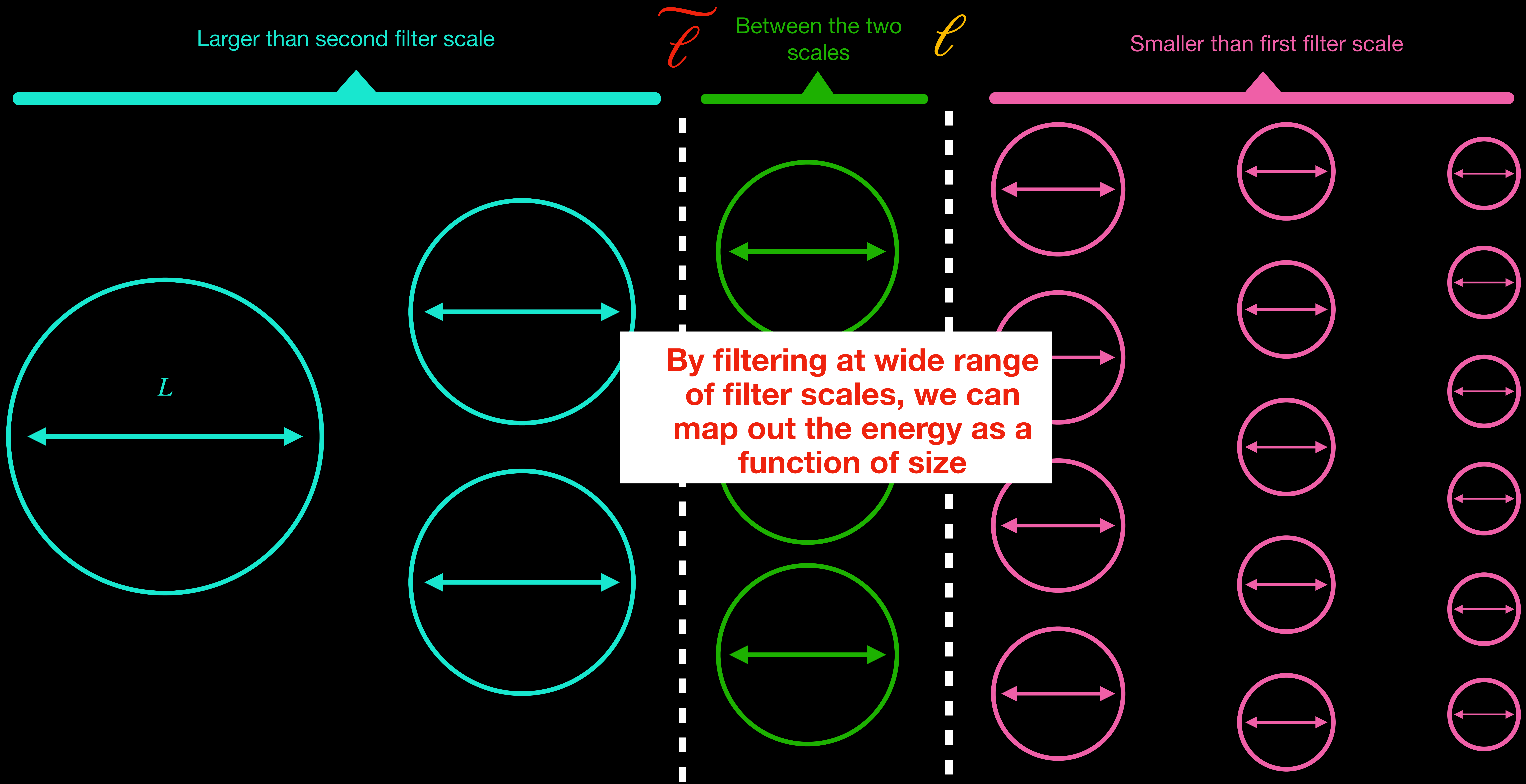
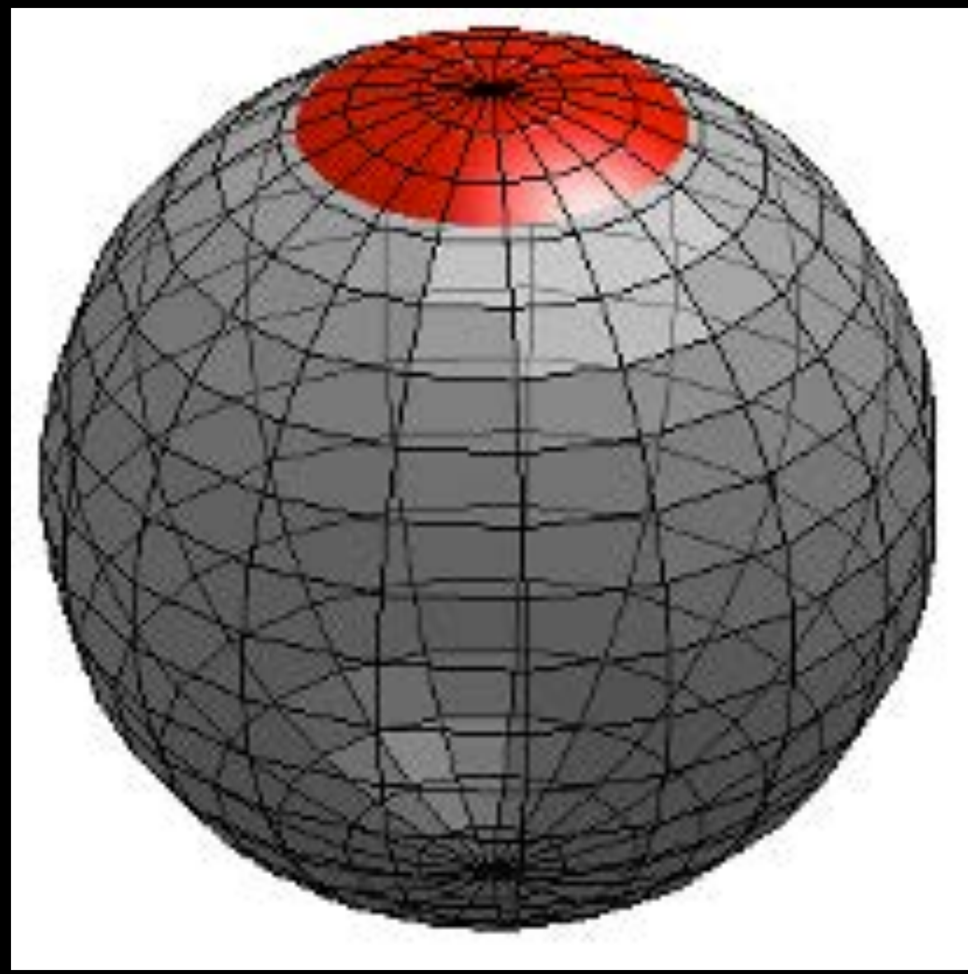


By filtering at wide range of filter scales, we can map out the energy as a function of size

Key Features of Coarse-Graining

- Systematically remove larger and larger scales, while maintaining information about where features are *in space*
- Can be applied directly / naturally to data on a sphere (i.e. geometry agnostic)
  - Distances measured along the sphere (i.e. geodesic)
- Can also be applied to the governing equations of motion
  - Study analyze both data and the physics
- Coarse-graining (when done carefully) commutes with derivatives



Coarse-graining: Compulsory Math Slide

Unfiltered
Eqs. of Motion

$$\frac{\partial}{\partial t}\vec{u} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho_0}\nabla P - f \times \vec{u} + \nu \nabla^2 \vec{u} + \frac{\rho}{\rho_0}\vec{g}$$

Filtered
Eqs. of Motion

$$\frac{\partial}{\partial t}\overline{\vec{u}} + \overline{\vec{u}} \cdot \nabla \overline{\vec{u}} = -\frac{1}{\rho_0}\nabla \overline{P} - f \times \overline{\vec{u}} - \nabla \cdot \overline{\tau}(\vec{u}, \vec{u}) + \nu \nabla^2 \overline{\vec{u}} + \frac{\overline{\rho}}{\rho_0}\vec{g}$$

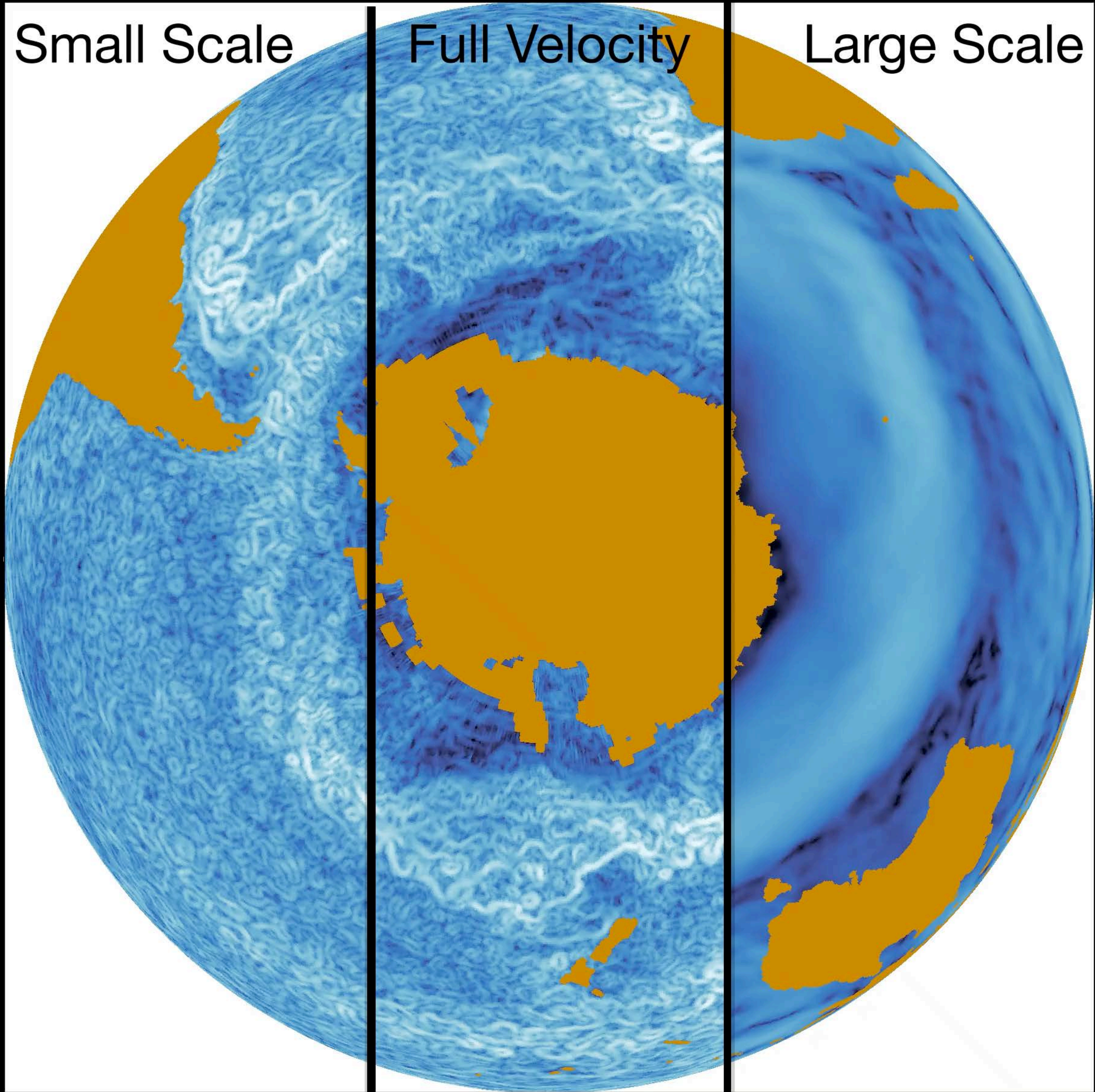
Large-Scale
Energy

$$\frac{\partial}{\partial t}\frac{\rho_0}{2}\left|\overline{\vec{u}}\right|^2 + \nabla \cdot J^{\text{transport}} = -\Pi - \rho_0 \nu \left|\nabla \overline{\vec{u}}\right|^2 + \overline{\rho} g \cdot \overline{\vec{u}}$$

So then, what can coarse-graining tell us about the ocean?

Can extract gyre-scale structures *without* time averaging

- Since we don't need to time average, we can make movies of scale-decomposed flows
- Small Scales: lots of spinning and twirling, but missing the main current
- Large Scales: very smooth, no spinning, but shows the current transporting water



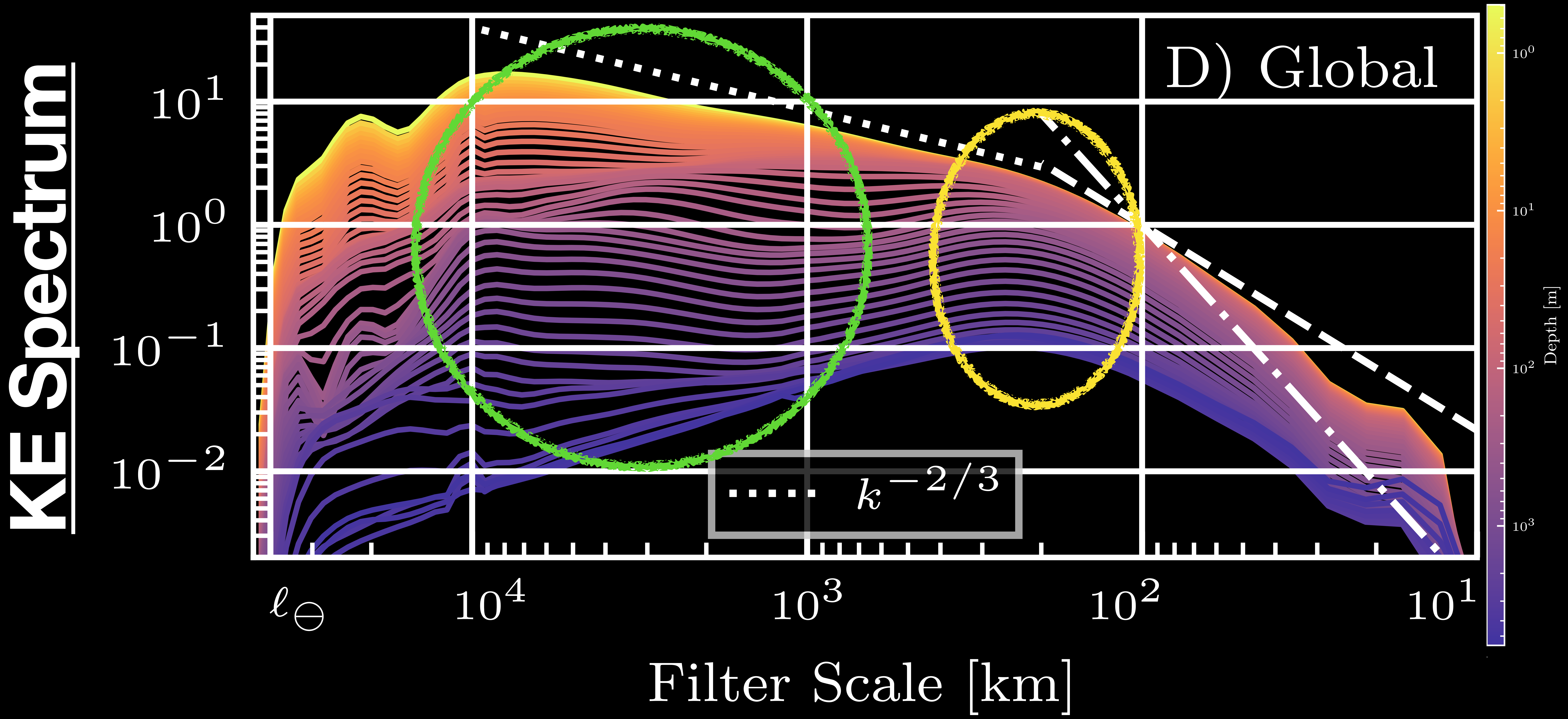
That was only one filter scale.

What if we want to study
many scales?

Sadek & Aluie, 2018

KE Spectra: SSH-Derived (Storer et al. 2022 NatComm)

- Surface trapping of gyre-scales
  - 10-fold decrease over 10s of metres
  - 100-1000-fold decrease in total
- Mesoscales do not lose energy in upper ~100m, retain larger percentage at depth

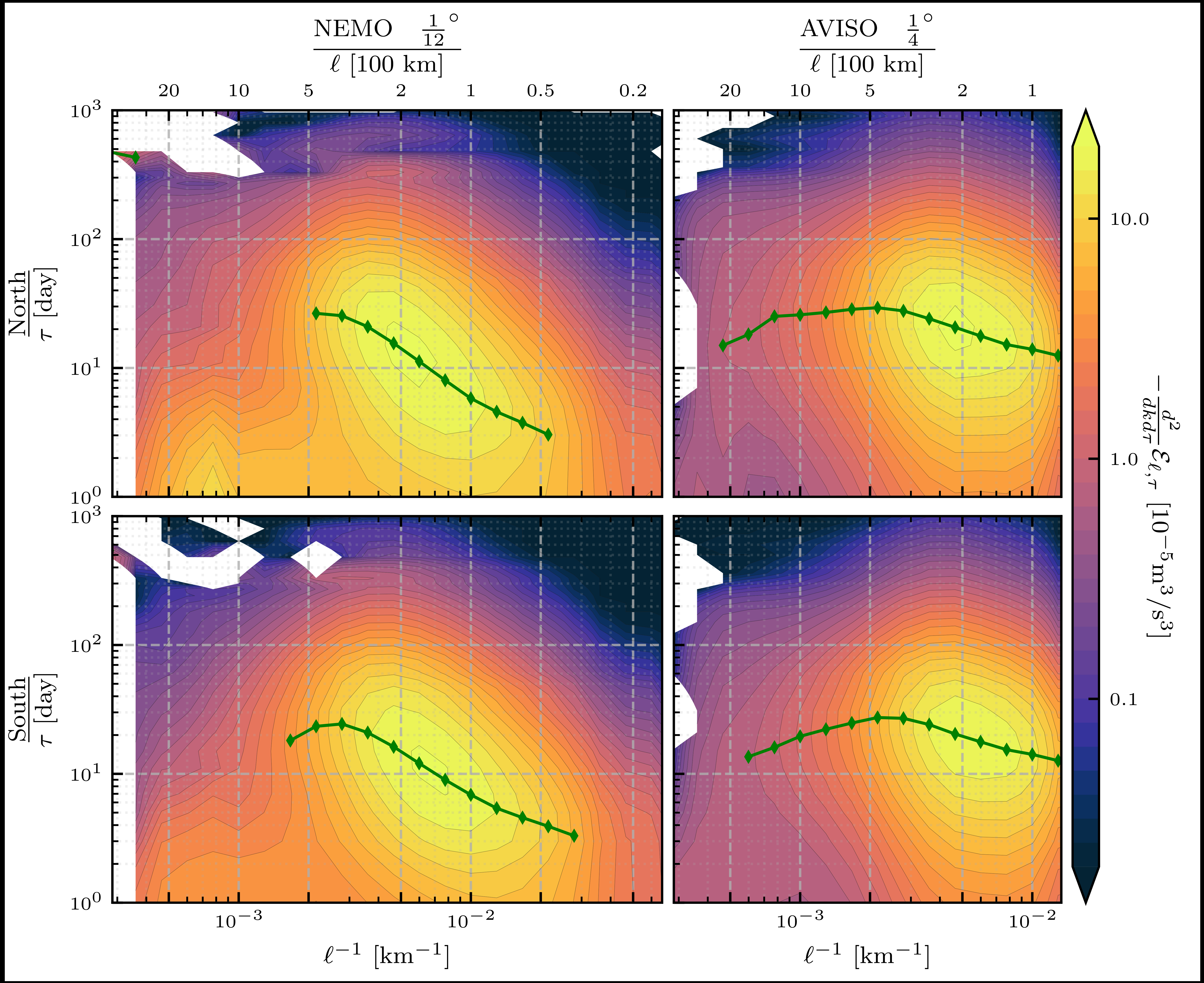


Coarse-Graining preserves the time signal, so can look at space-time spectra

(i.e. what notes are played *and* how long they are player)

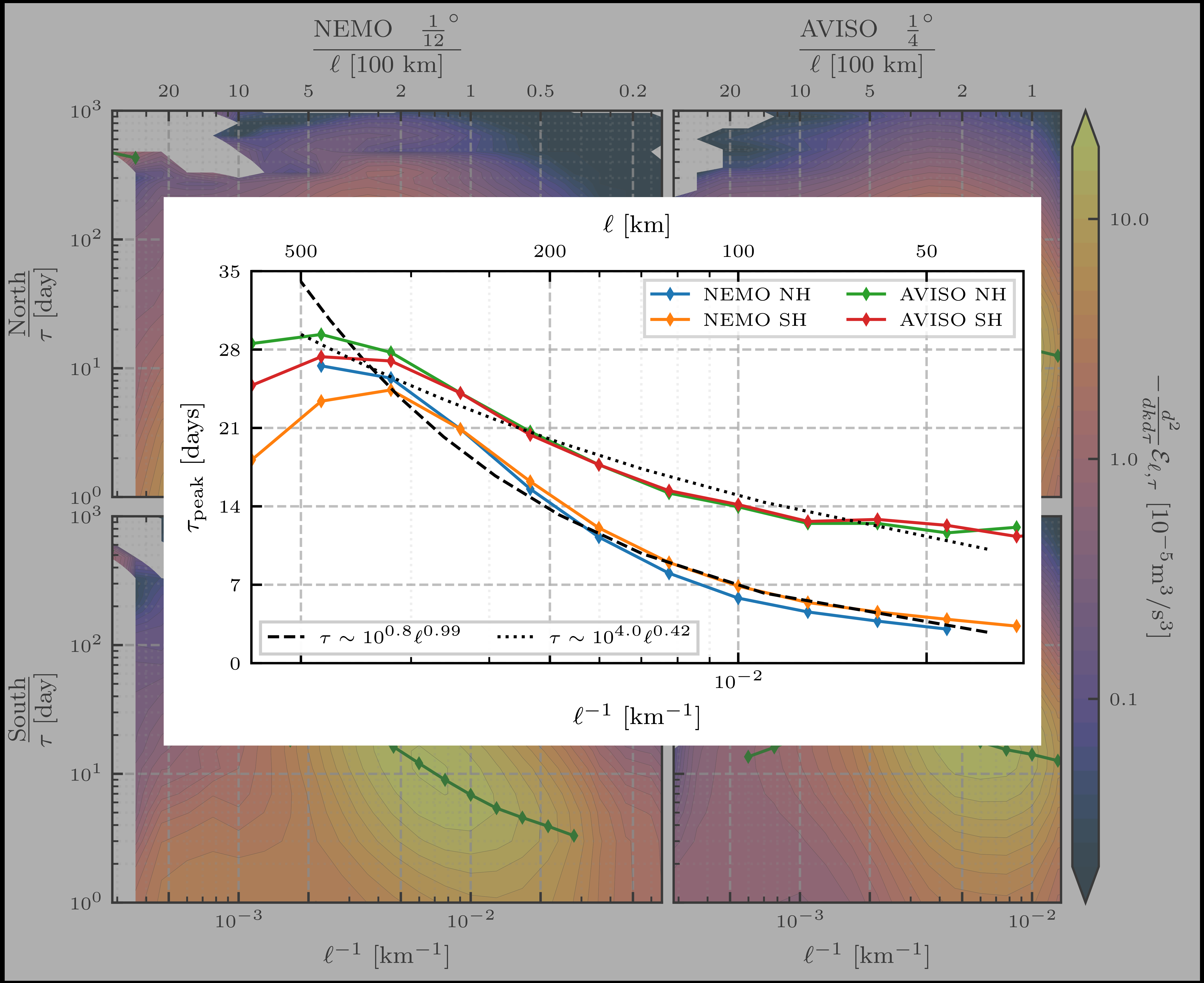
Space-Time Spectra

- Energy peaks around ~200km and 2 weeks
- AVISO uses time averaging to build full maps
    - can see loss of short time-scale energy
- High-frequency large-scale energy signal may be pressure loading



Space-Time Spectra

- Energy peaks around ~200km and 2 weeks
- AVISO uses time averaging to build full maps
    - can see loss of short time-scale energy
- High-frequency large-scale energy signal may be pressure loading
- Peak time-scale roughly proportional to length-scale



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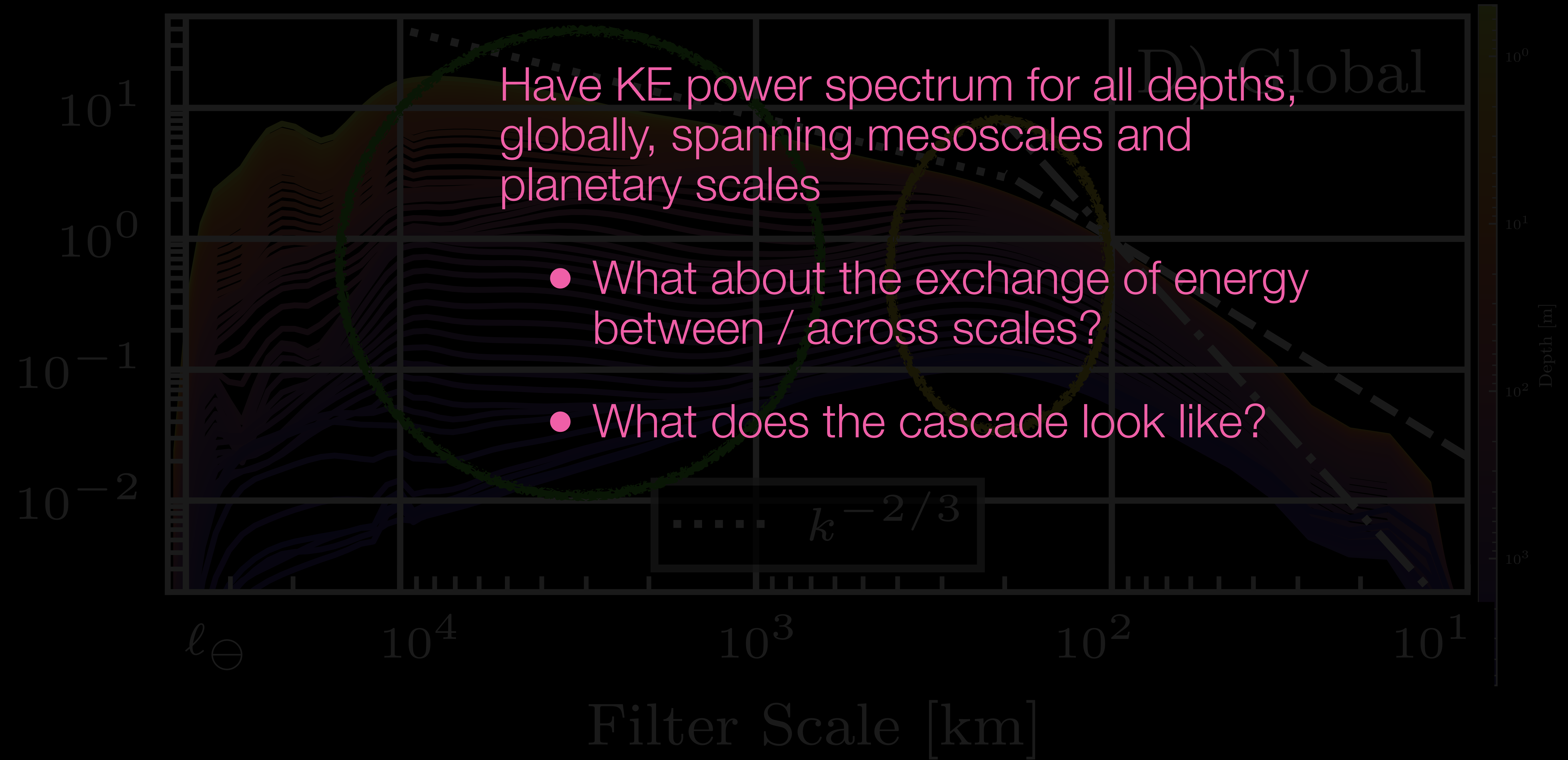
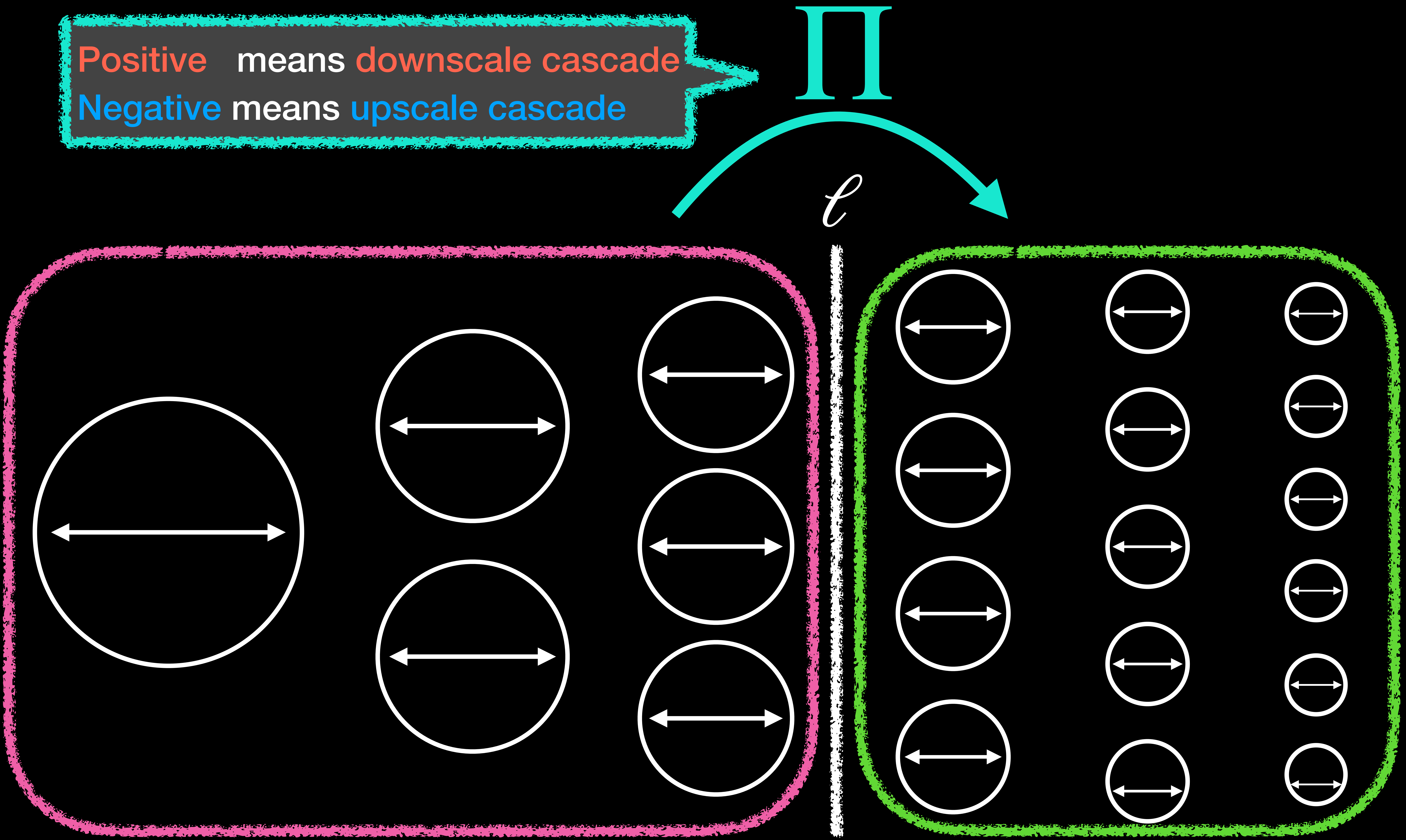


Have KE power spectrum for all depths, globally, spanning mesoscales and planetary scales

- What about the exchange of energy between / across scales?
- What does the cascade look like?

Energy Cascade (Π)

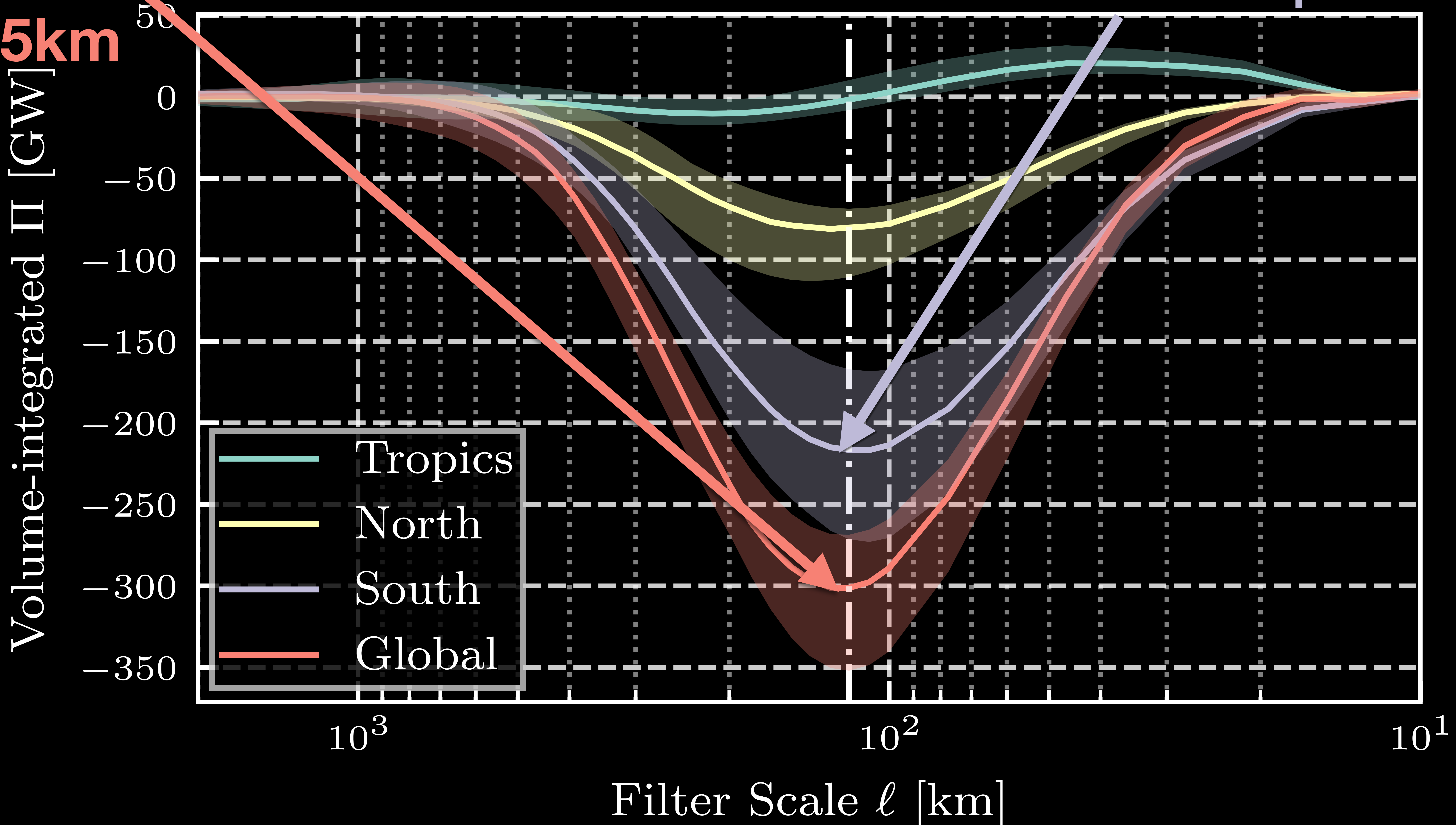


KE Cascade: Volume Integrated

~75% of mesoscale cascade occurs in Southern Hemisphere

Global integrated cascade peaks at ~300GW at ~125km

[Winds input ~1TW to ocean circulation]



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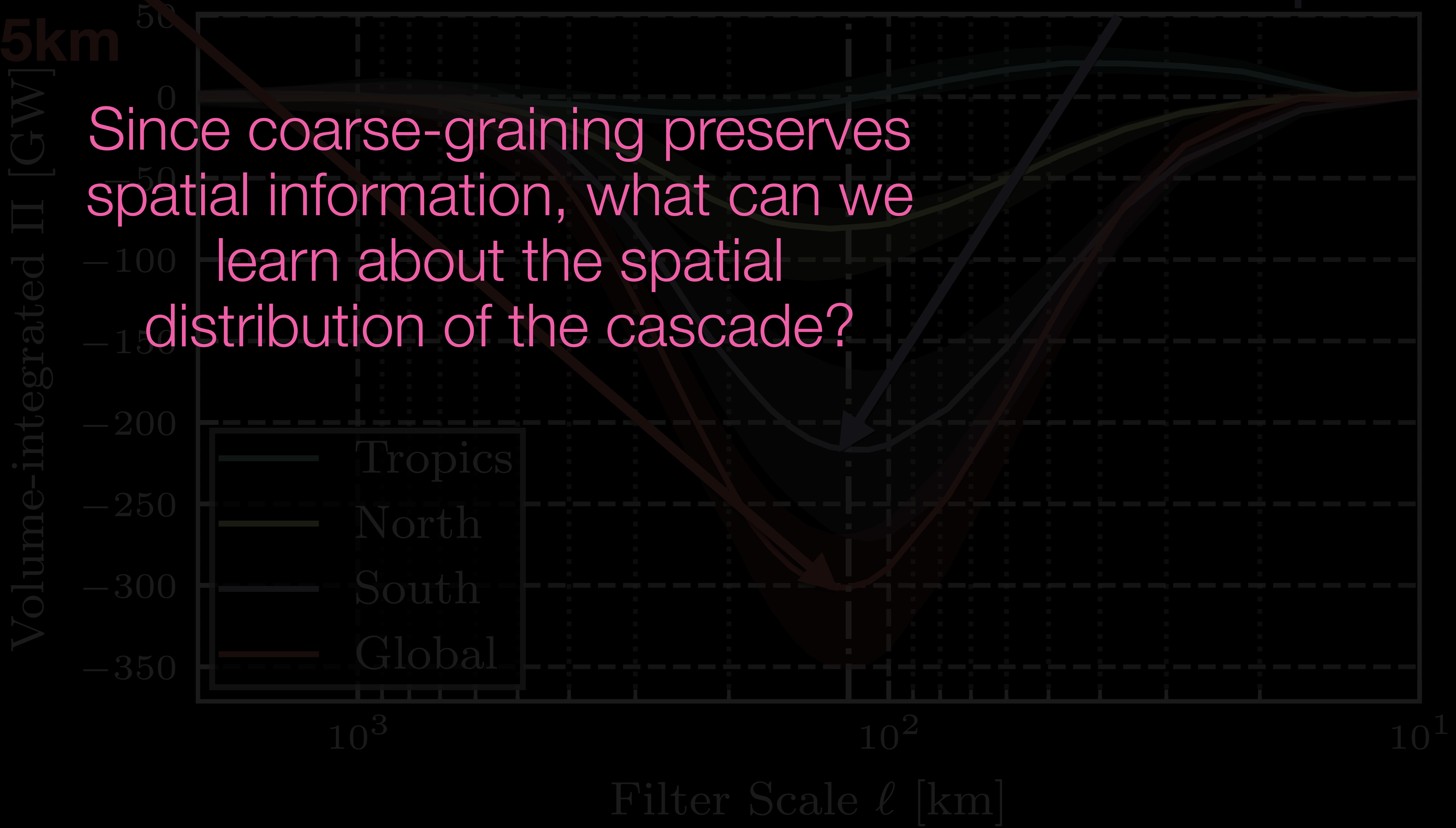
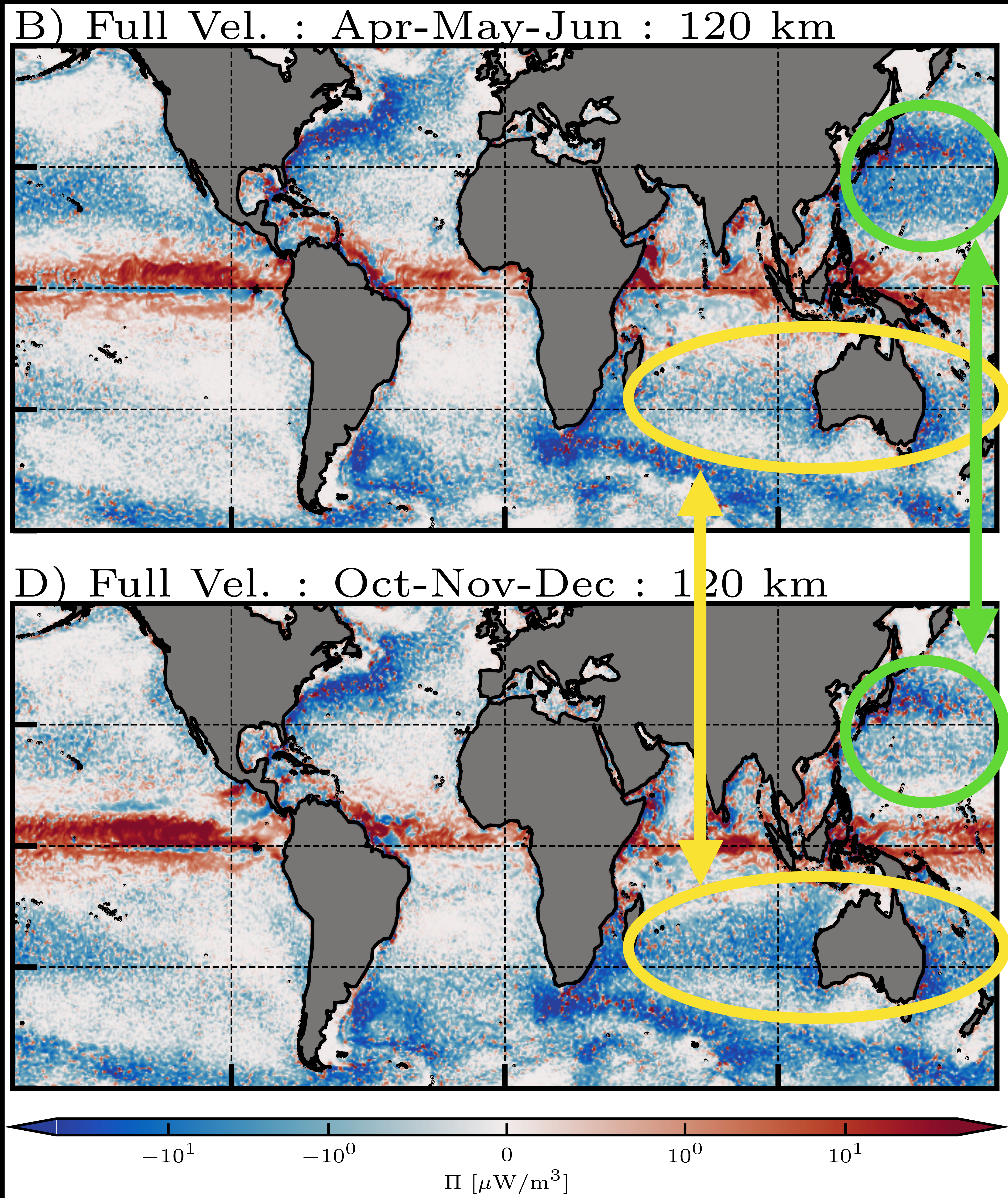


Since coarse-graining preserves spatial information, what can we learn about the spatial distribution of the cascade?

Cascade through ell = 120 km

Mesoscale Inverse Cascade strengthens / expands in local spring



Cascade through ell = 1000 km

Imprint of atmospheric cells visible in maps

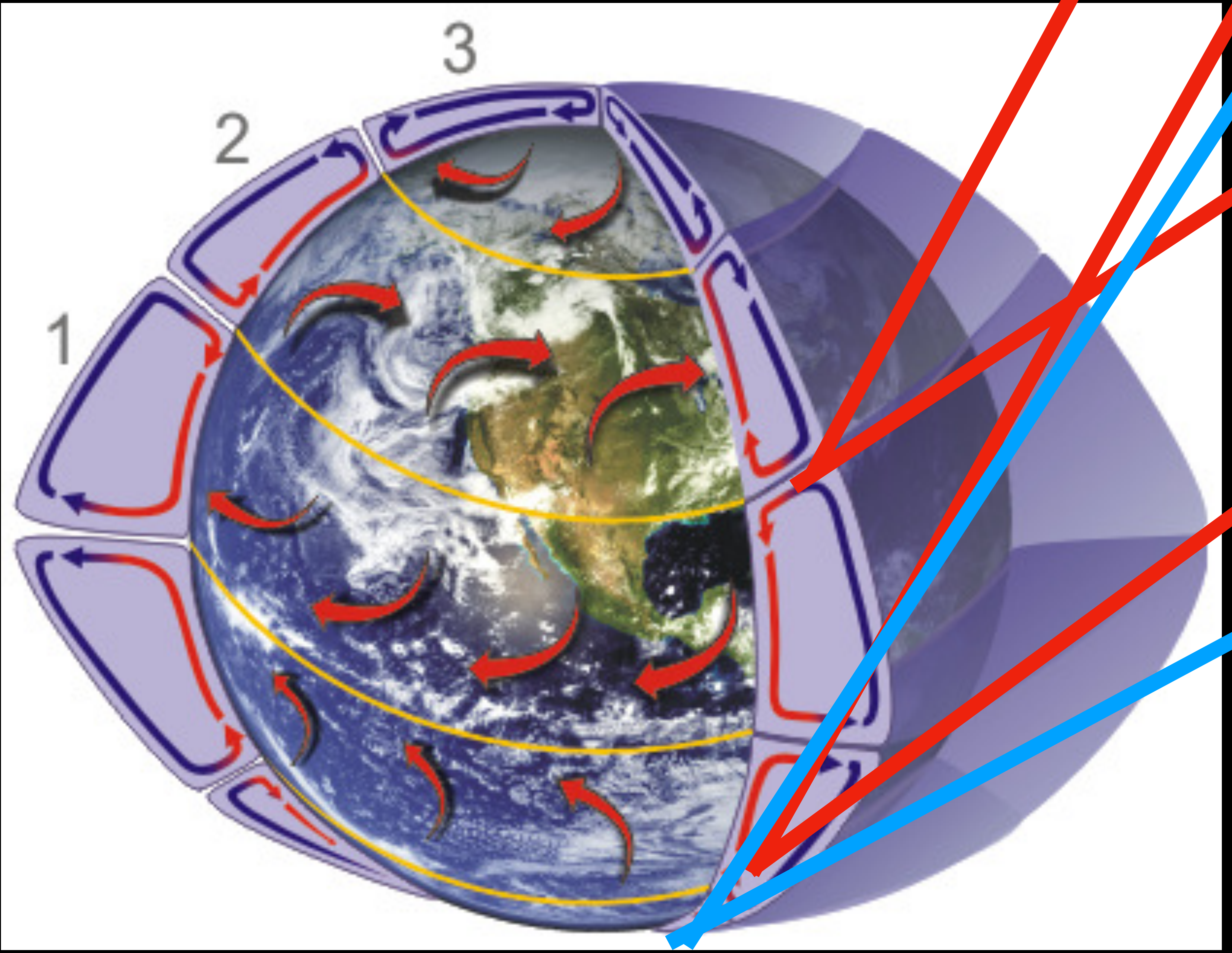
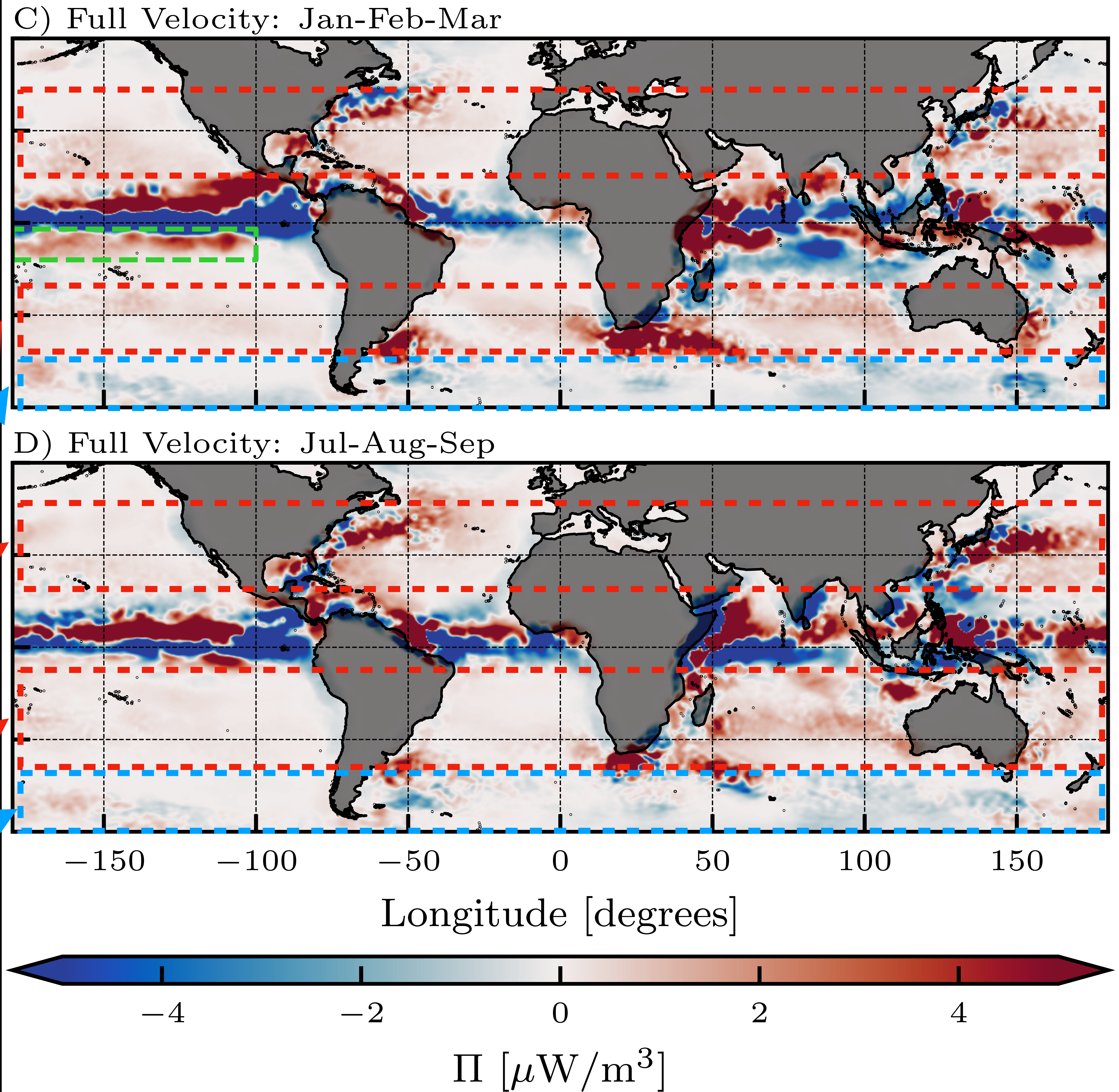
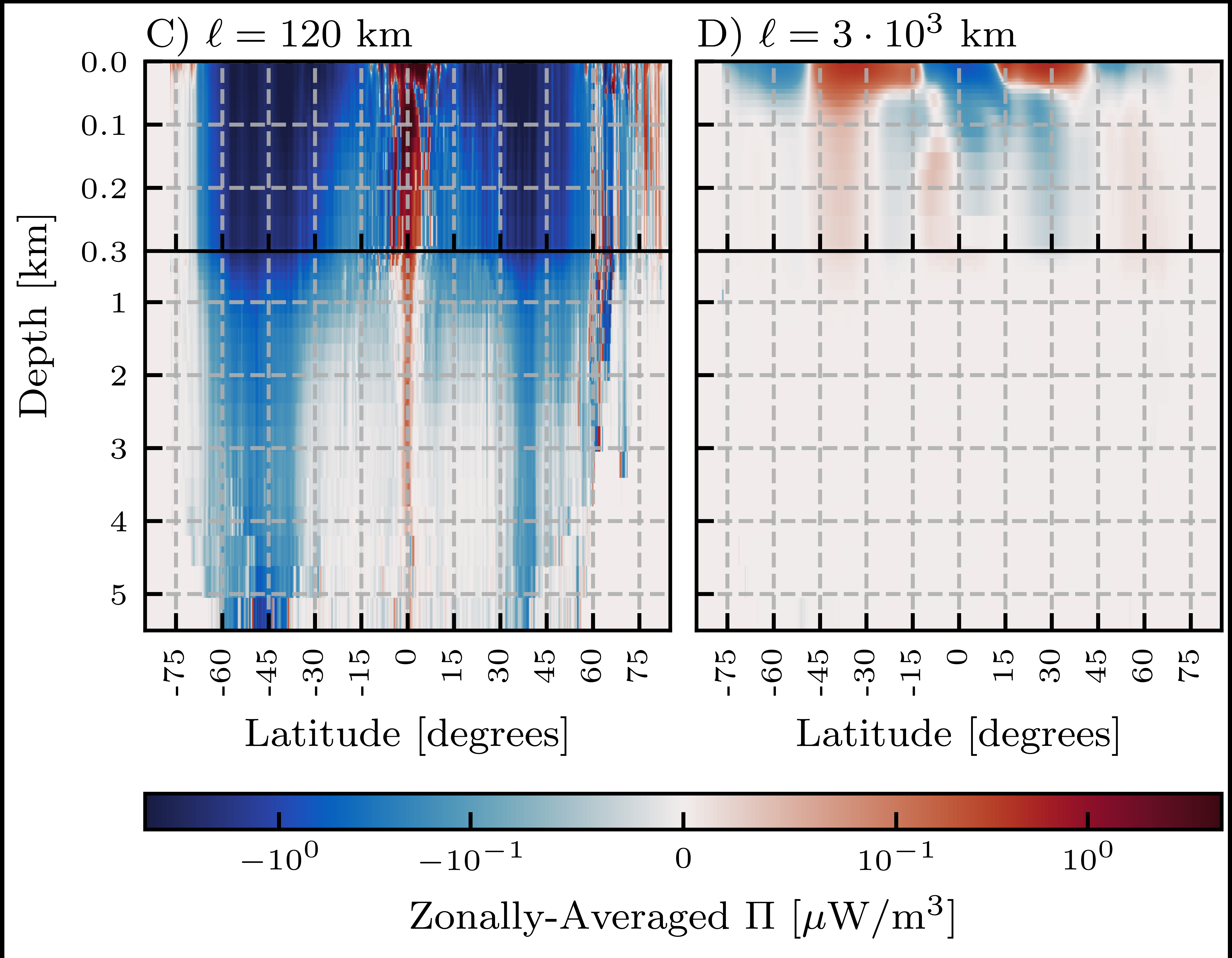


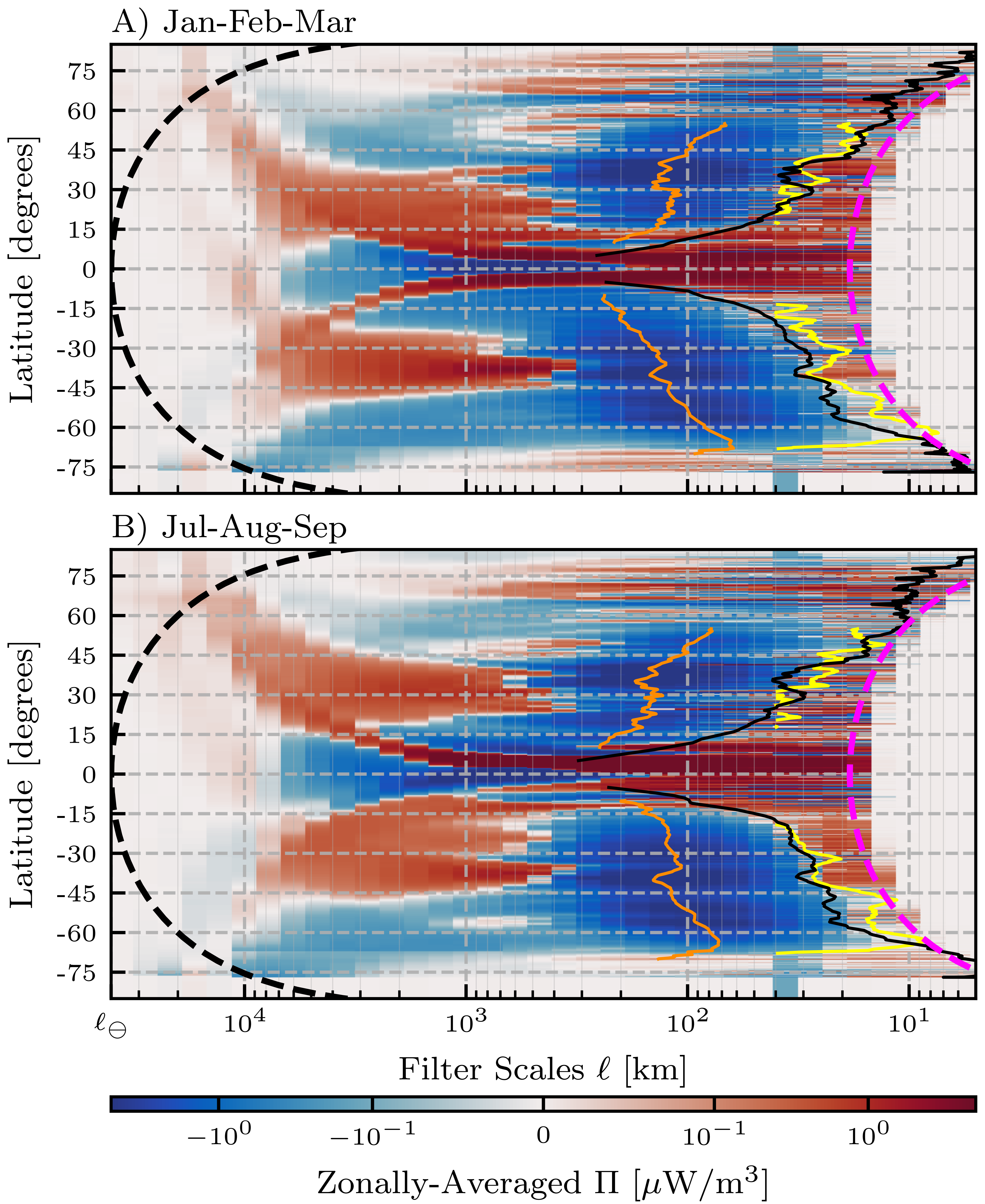
Image Source: National Weather Service



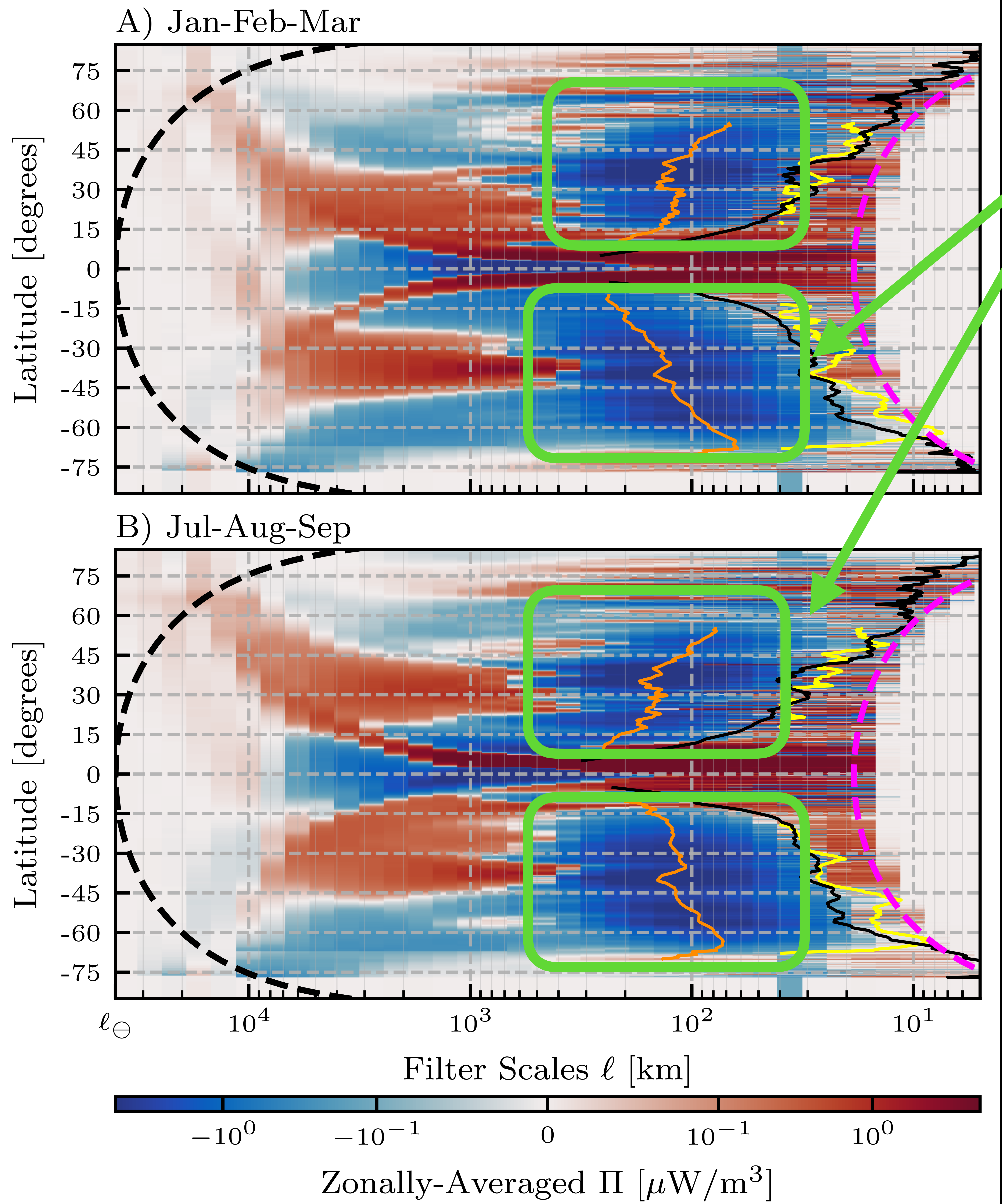
Mesoscale Inverse Cascade Spans Entire Water Column

Atmospheric Cell-induced Cascade only upper ~50m (i.e. Ekman layer)



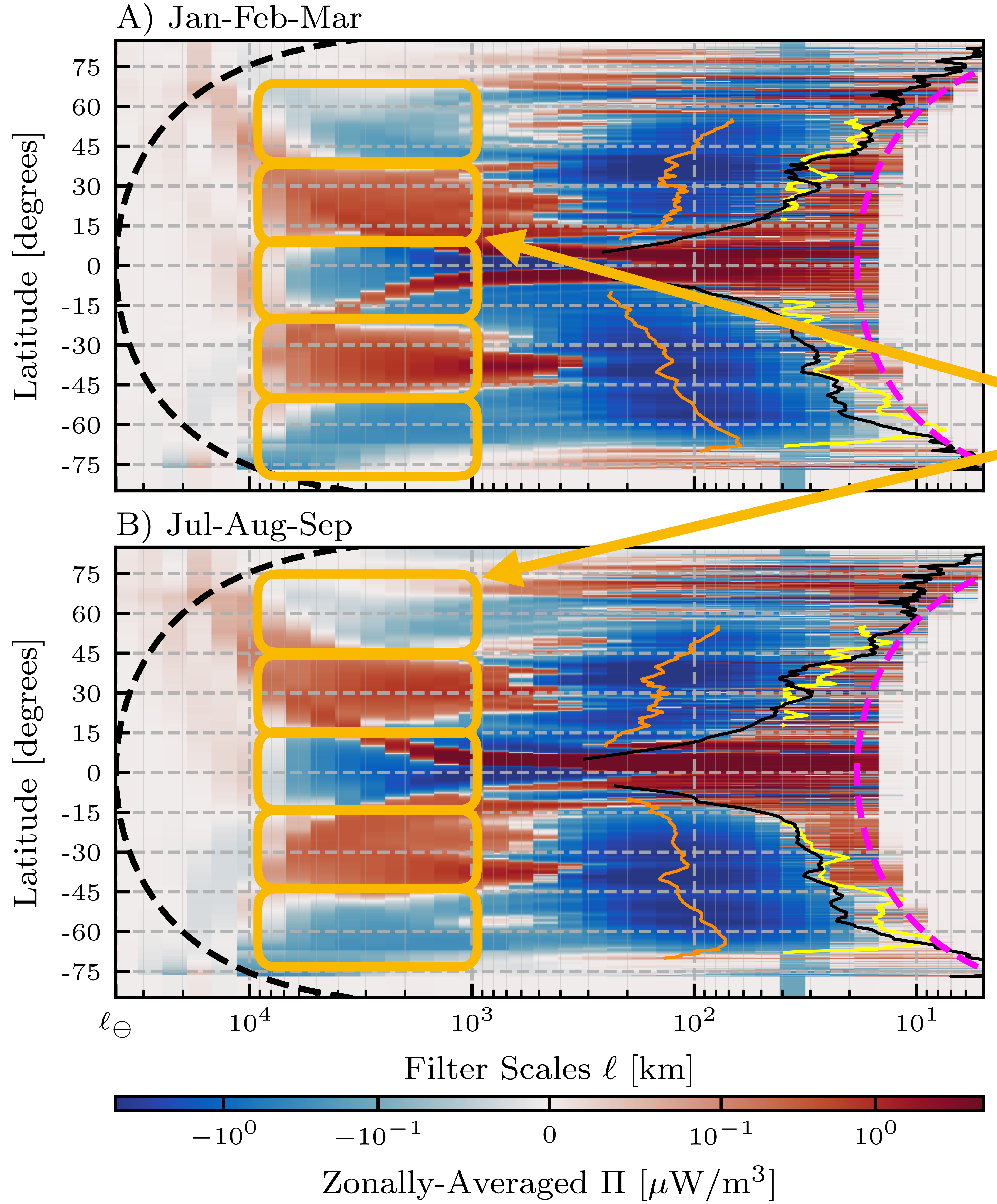


A) Jan-Feb-Mar
B) Jul-Aug-Sep
Latitude [degrees]
75
60
45
30
15
0
-15
-30
-45
-60
-75
$\ell_\ominus$
10^4
10^3
10^2
10^1
Filter Scales ℓ [km]
-10^0
-10^{-1}
0
10^{-1}
10^0
Zonally-Averaged Π [μW/m^3]



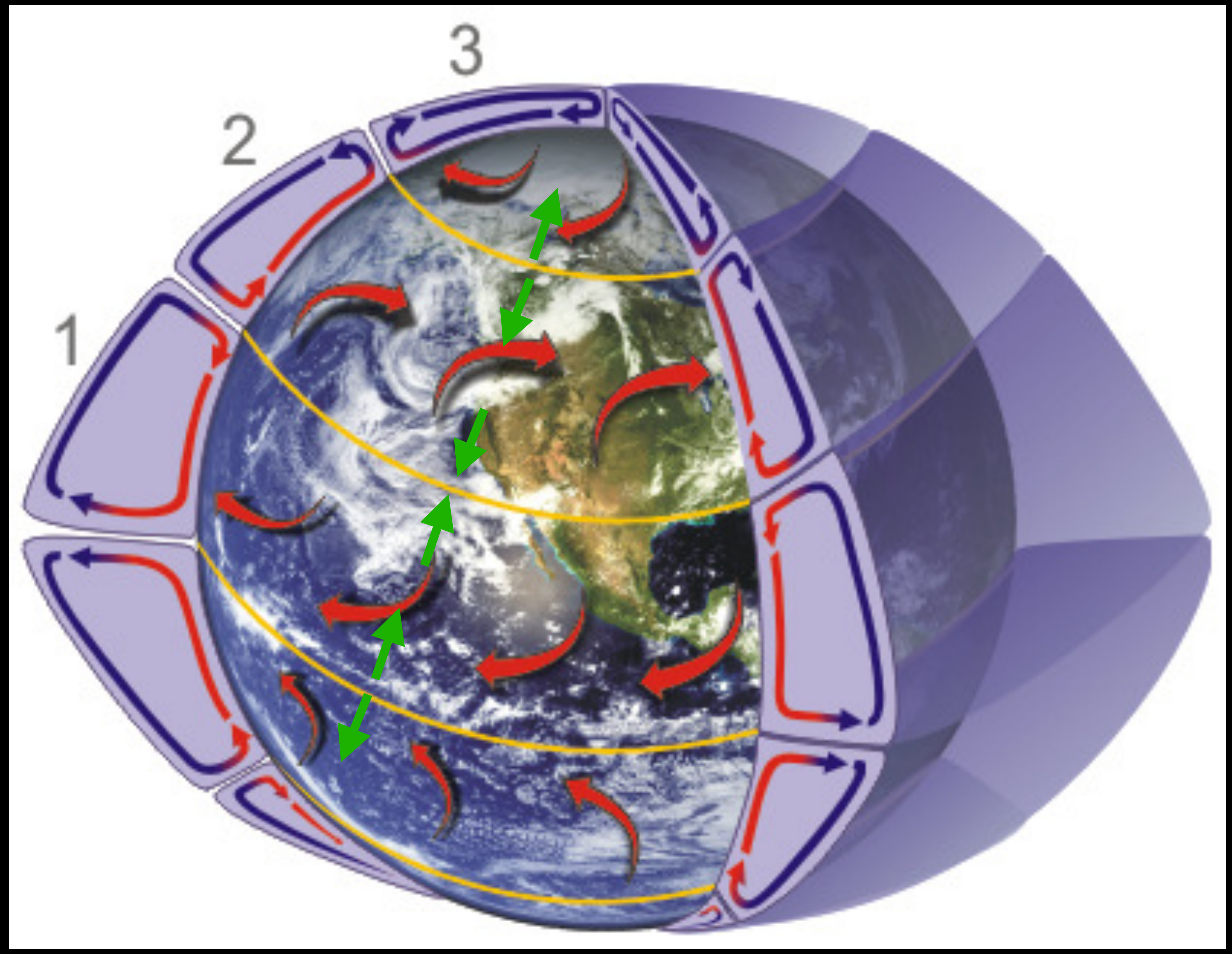
Mesoscale Inverse Cascade

Length-scale of dominant cascade decreases towards the poles



Mesoscale Inverse Cascade

Imprint of Hadley, Ferrel, and Polar Cells, through Ekman divergence / convergence



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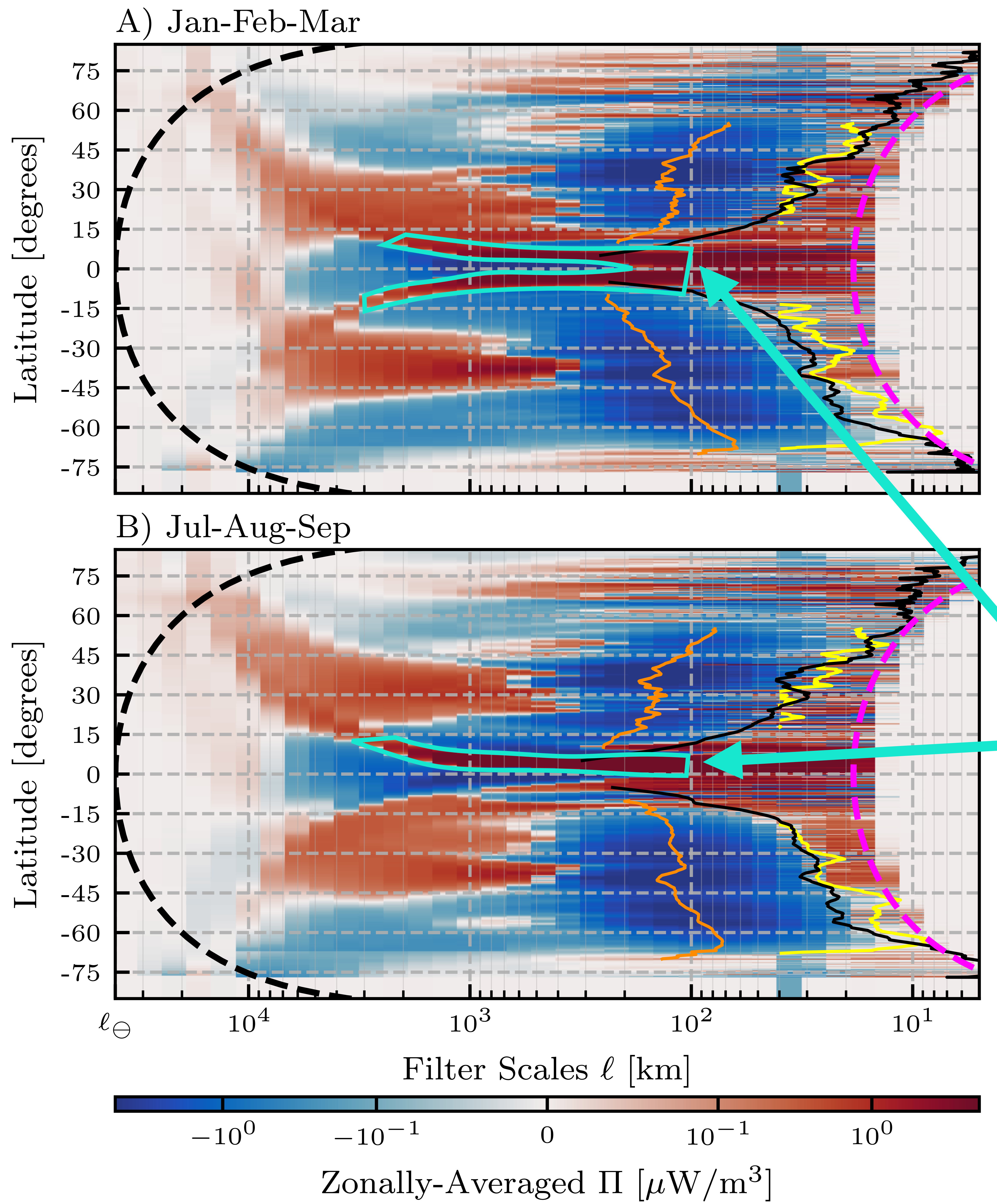


Mesoscale Inverse Cascade

Imprint of Hadley, Ferrel, and Polar Cells, through Ekman divergence / convergence

Narrow Down-scale Branches Near Equator : Inter-Tropical Convergence Zone (ITCZ)

We’ve looked at KE spectra and KE cascades

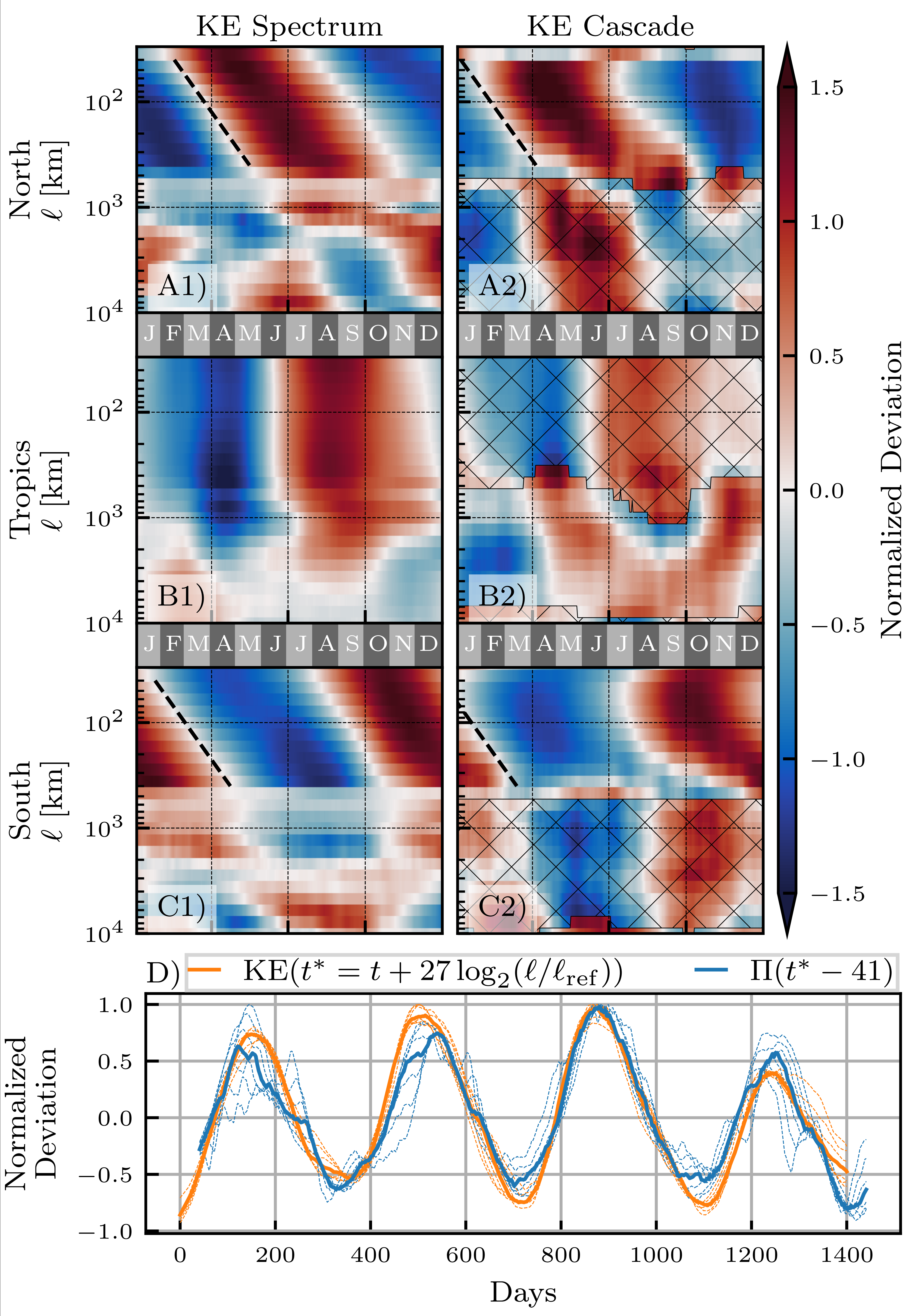
but what connections can we find between them?

Blue

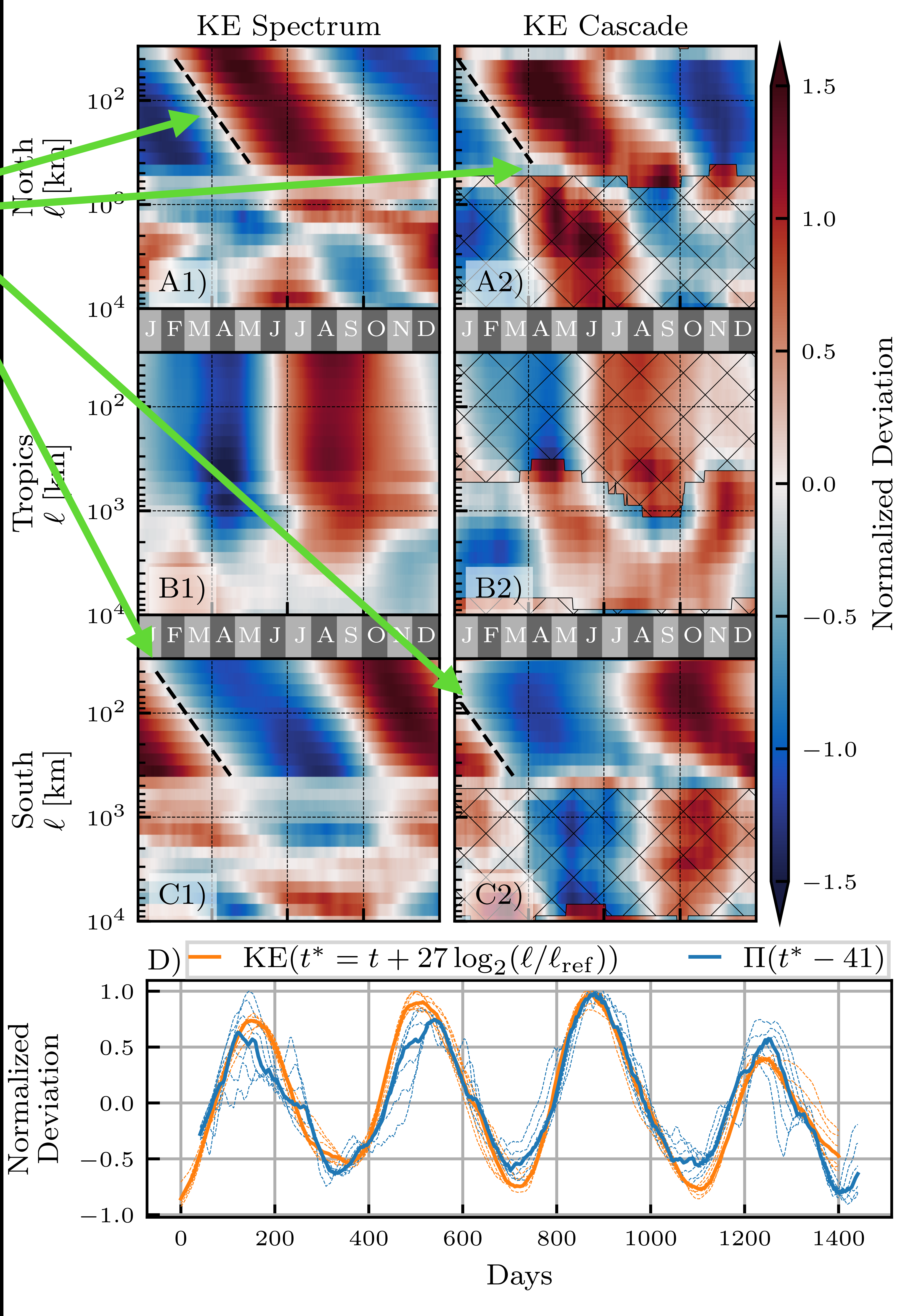
- seasonally low KE
- seasonally low Π magnitude

Red

- seasonally high KE
- seasonally high Π magnitude



- For $50 \text{ km} \lesssim \ell \lesssim 500 \text{ km}$, seasonal cycle of larger scales happens later than smaller scale
  - ~27 days per octave
  - i.e. if ℓ has seasonal max KE today, 2ℓ will have seasonal max KE in ~4 weeks
- Seasonal cycle of Π occurs ~41 days earlier than KE
  - i.e. if if ℓ has seasonal max Π today, ℓ will have seasonal max KE in ~41 days



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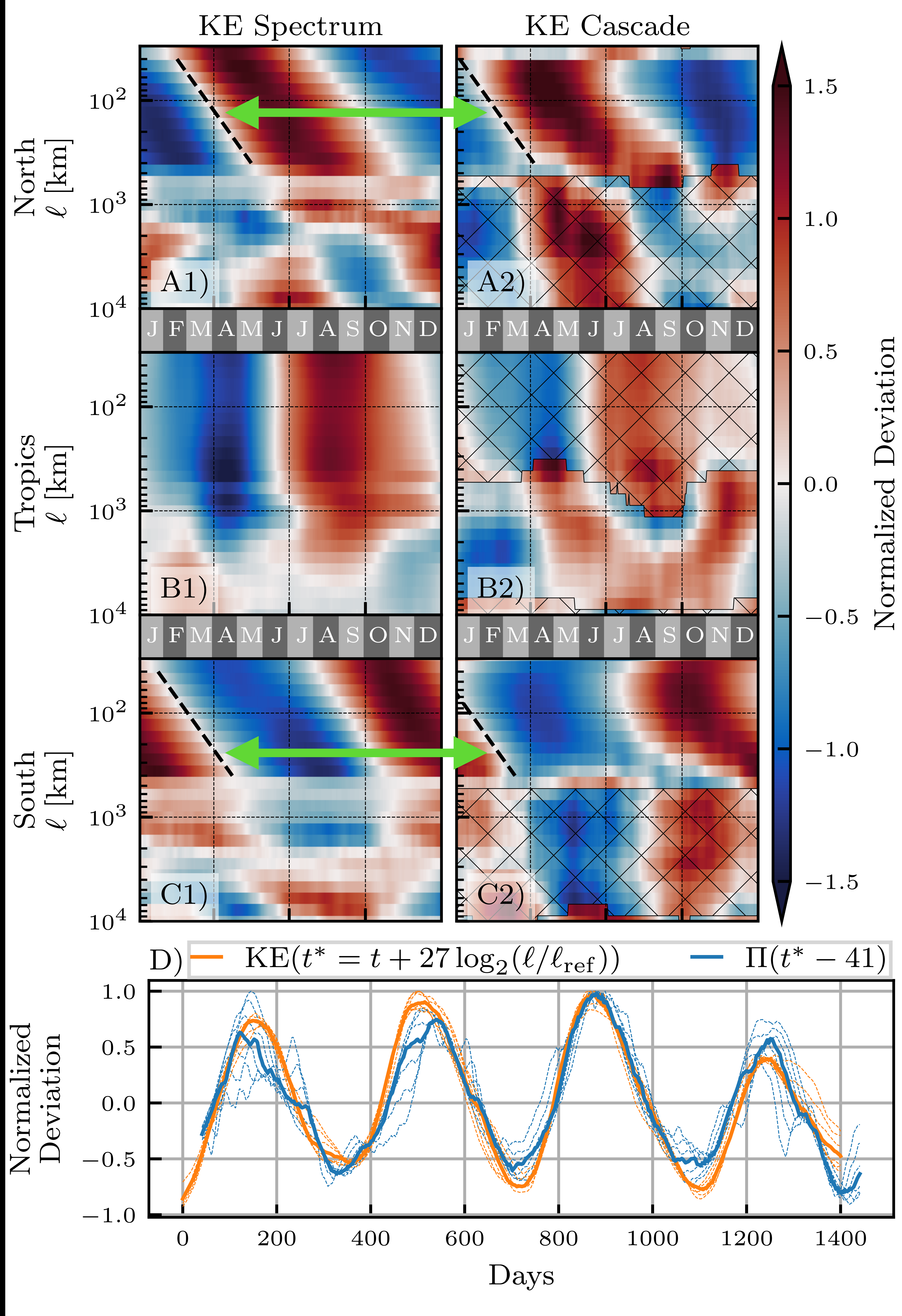
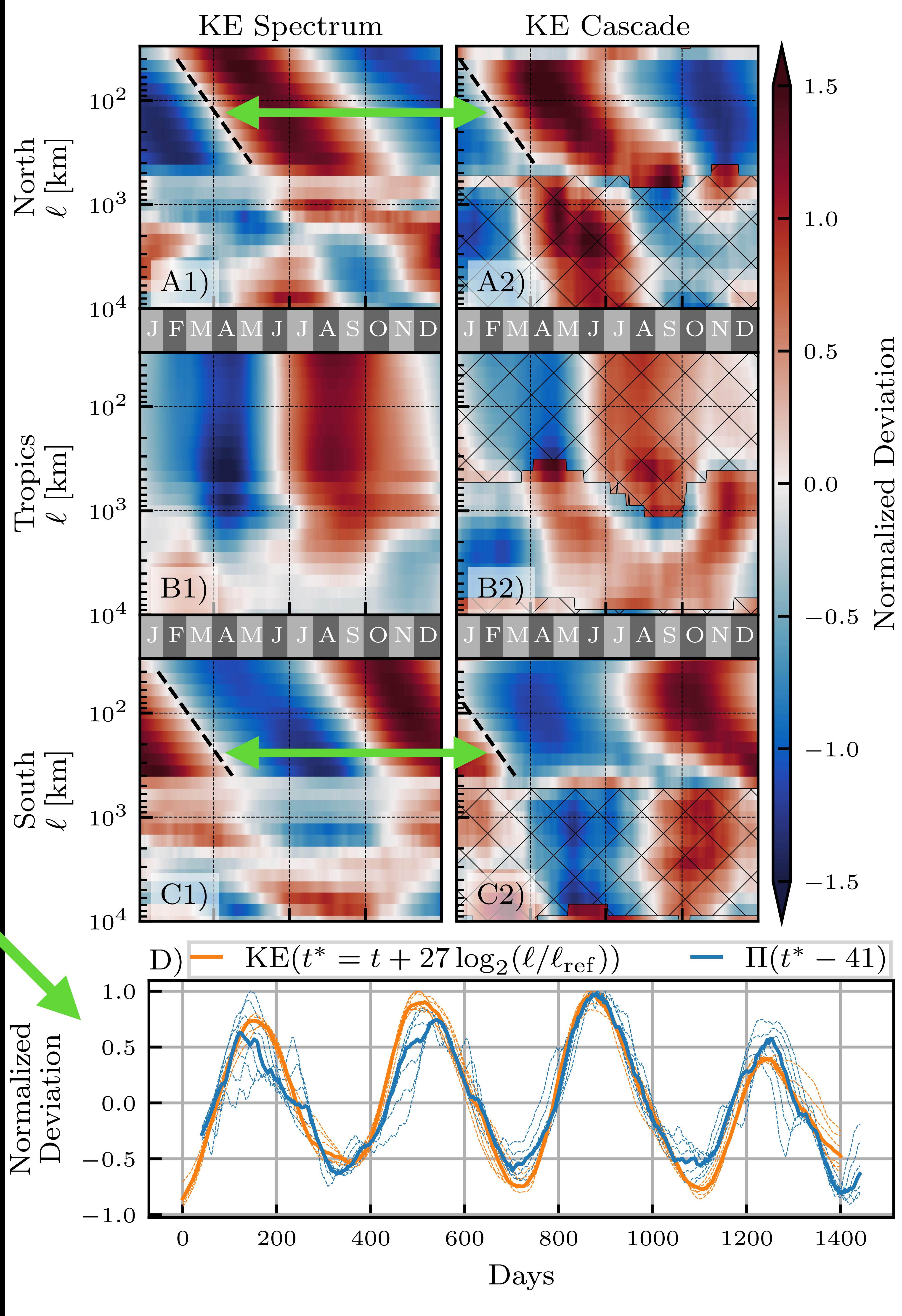


- If we phase adjust the KE and Π signal at each scale, they collapse onto the same curve



- Seasonal cycle of Π occurs ~41 days earlier than KE

$\frac{d}{dt}$[KE Spectrum] ⟵ ~50 Days ⟶ Π ⟵ ~41 Days ⟶ KE Spectrum

- ~27 days per octave

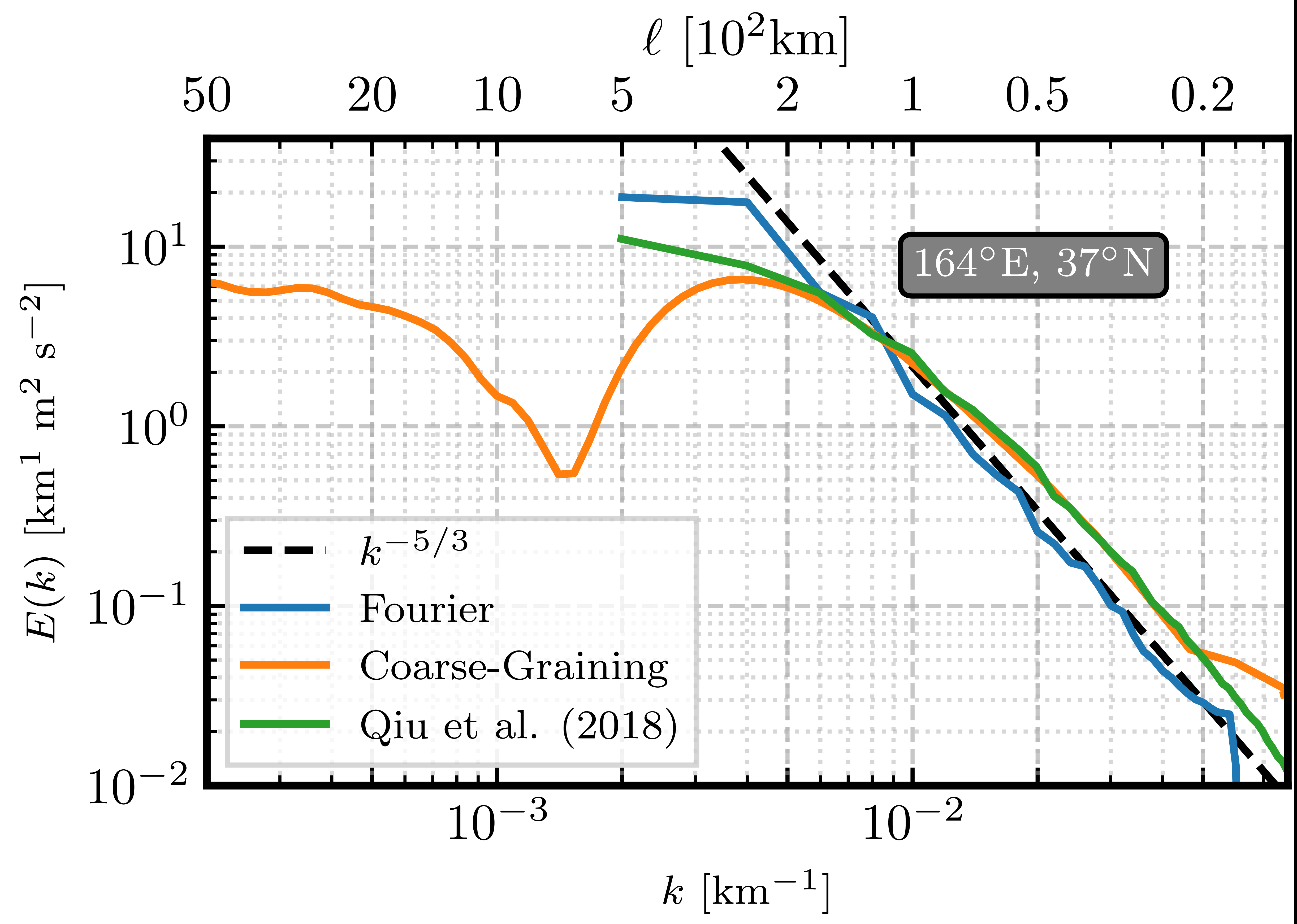
$$\ell_{\frac{d}{dt}\mathrm{KE}} \approx 3.8\, \ell_{\Pi} \qquad\qquad \ell_{\mathrm{KE}} \approx \frac{1}{3}\, \ell_{\Pi}$$

- Most energetic scale ~ 3 times smaller than cascade scale
- Fastest growing scale ~4 times larger than cascade scale

We've seen what we can do with coarse-graining.

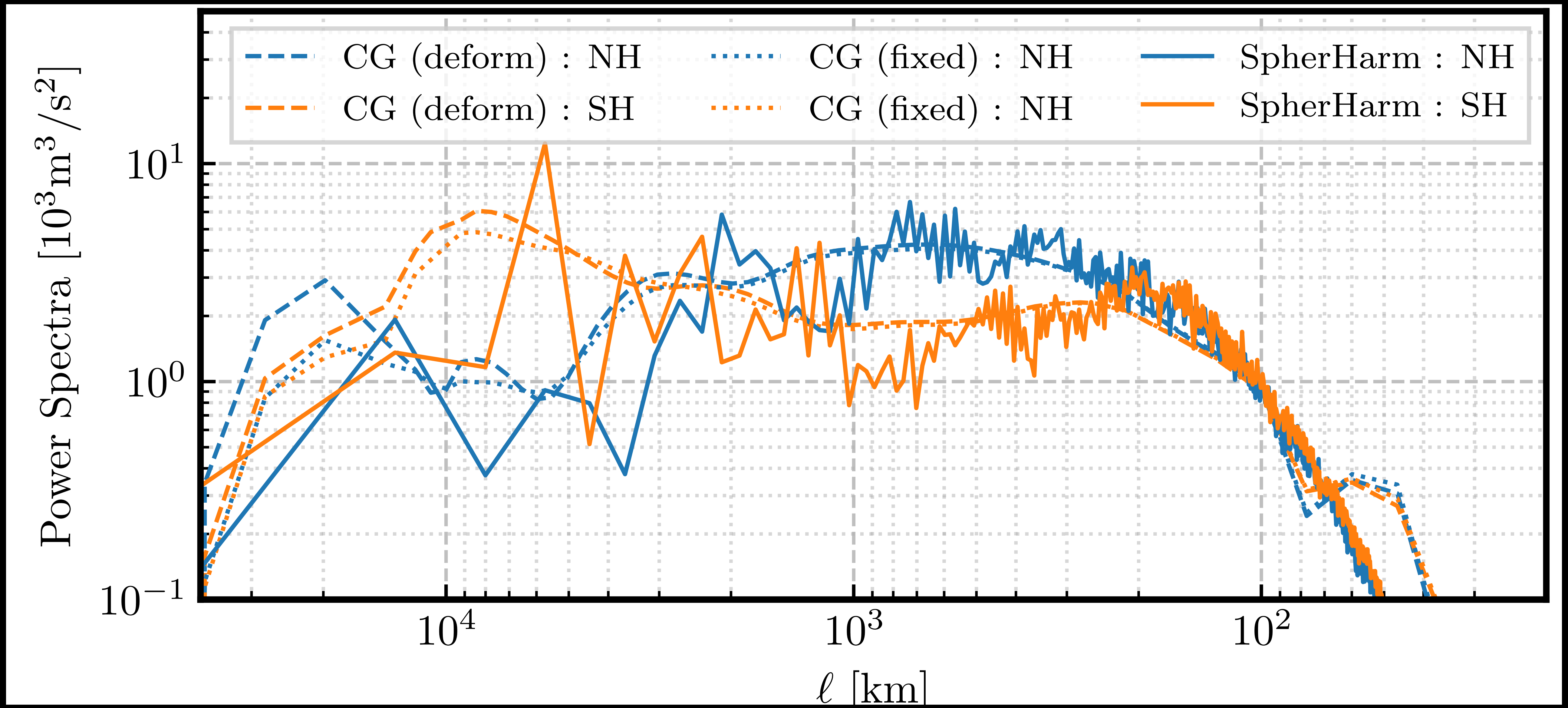
How does this compare with other methods?

Comparing Coarse-Graining with Fourier Transforms



- Where Fourier methods are valid, the two agree well
- Coarse-graining not limited to a "box", can go to larger scales

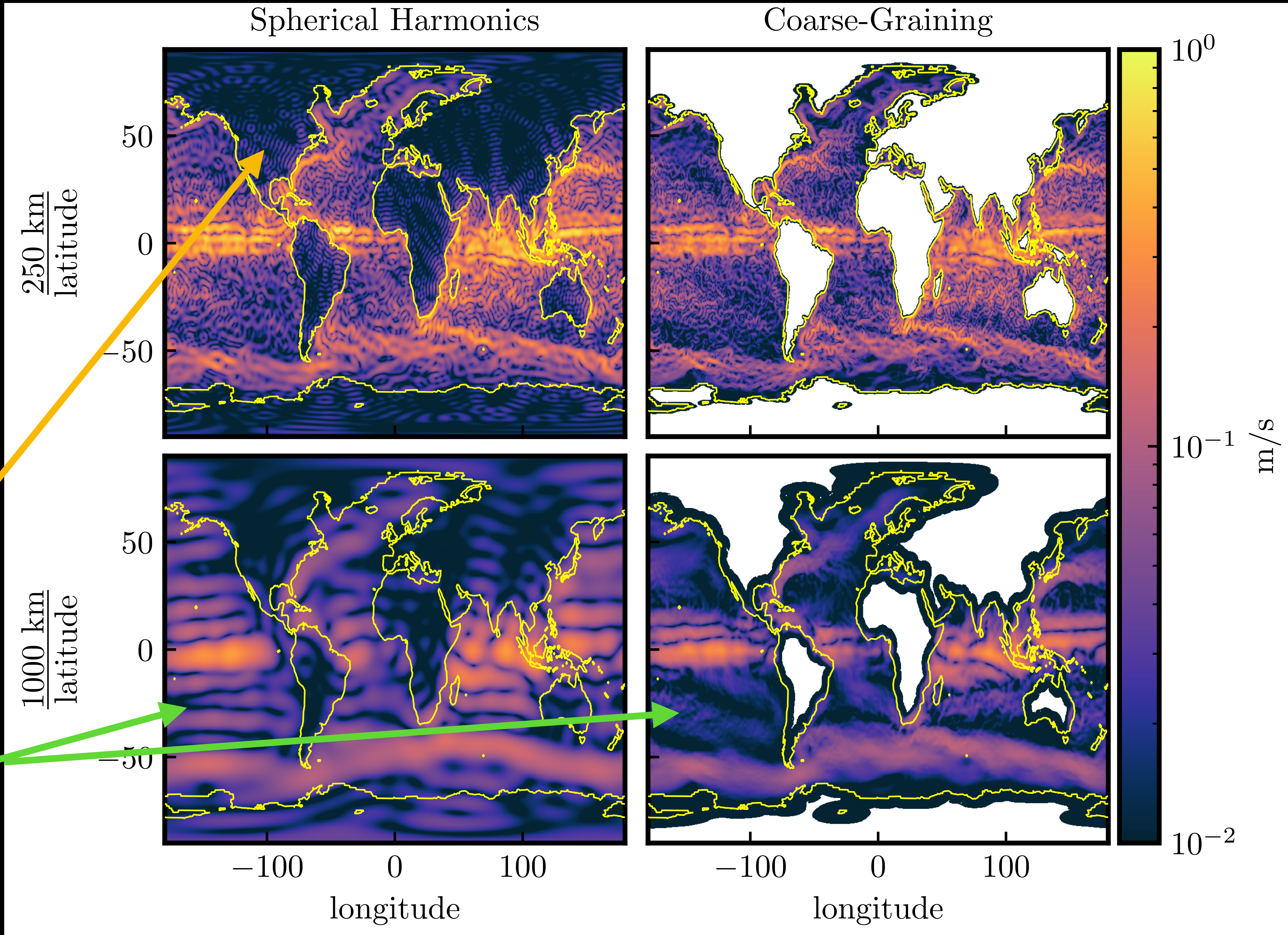
Comparing Coarse-Graining with Spherical Harmonics



- Spherical Harmonics and Coarse-Graining generally agree well
- Coarse-graining allows you to choose the length scales / wavenumbers

Comparing Coarse-Graining with Spherical Harmonics

- Coarse-graining
  - non-zero values only extend $\ell/2$ into land (typically low magnitudes)
- Spherical Harmonics
  - non-zero values throughout land areas
  - 'ringing' also fills in low-energy ocean areas



Reynolds Decomposition

- More than half of the time-mean energy is in scales smaller than 500km
- Highlights importance of standing eddies

		Full Velocity	Time-Mean	Time-Varying
% of Energy in Scales < 500km	NH	91	71	97
	SH	90	57	98

Outro Slides

- Coarse-graining is gaining traction as a powerful tool for scale analysis of complex systems
  - Rai et al. (2021), Srinivasan et al. (2022), Khatri et al. (2023, submitted)
- FlowSieve (Storer & Aluie, 2023) a publicly available codebase for scale analysis
- Exciting new avenues for analysis!
  - Ocean-Atmosphere interaction [Ekman transport, cascades, spiral]
  - Studying (quantitatively!) the temporal evolution of large-scale systems
  - Analyzing scalar distributions